MA347 – HW18

Jonathan Lam

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1. Find all abelian groups of order 1160 up to isomorphism.

$$1160 = (2^3)(5)(29)$$

There are three groups of order 8 up to isomorphism: $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$, $\mathbb{Z}_{2^2} \oplus \mathbb{Z}_2$, and $\mathbb{Z}_{2^3}$. There is only one group of order 5 and one group of order 29 up to isomorphism: the cyclic groups $\mathbb{Z}_5$ and $\mathbb{Z}_{29}$, respectively.

Thus there are three groups with order 1160 up to isomorphism.

$$\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_{29} \simeq \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_{290}$$
$$\mathbb{Z}_2 \oplus \mathbb{Z}_{2^2} \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_{29} \simeq \mathbb{Z}_2 \oplus \mathbb{Z}_{580}$$
$$\mathbb{Z}_{2^3} \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_{29} \simeq \mathbb{Z}_{1160}$$

[It is easy to see that any group of order 1160 falls into one of these sets (due to the factorization of 1160), and that these isomorphism classes are mutually distinct (due to the theorem from class).]

2. Find the invariant factors and the elementary divisors of:

(a) $A = \mathbb{Z}_{10} \oplus \mathbb{Z}_{20} \oplus \mathbb{Z}_{30} \oplus \mathbb{Z}_{40}$

$$\begin{aligned}
\mathbb{Z}_{10} &\simeq \mathbb{Z}_2 \oplus \mathbb{Z}_5 \\
\mathbb{Z}_{20} &\simeq \mathbb{Z}_{2^2} \oplus \mathbb{Z}_5 \\
\mathbb{Z}_{30} &\simeq \mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_5 \\
\mathbb{Z}_{40} &\simeq \mathbb{Z}_{2^3} \oplus \mathbb{Z}_5
\end{aligned}$$

thus

$$A \simeq \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_{2^2} \oplus \mathbb{Z}_{2^3} \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_5$$

and $\{2, 2, 4, 8, 3, 5, 5, 5, 5\}$ are the elementary divisors of A. Using the visual method from class for invariant factors:

2	2	2^2	2^3
			3
5	5	5	5
10	10	20	120

which are the four invariant factors of A.

(b) $B = \mathbb{Z}_{12} \oplus \mathbb{Z}_{30} \oplus \mathbb{Z}_{100} \oplus \mathbb{Z}_{240}$

$$\begin{aligned}
\mathbb{Z}_{12} &\simeq \mathbb{Z}_{2^2} \oplus \mathbb{Z}_3 \\
\mathbb{Z}_{30} &\simeq \mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_5 \\
\mathbb{Z}_{100} &\simeq \mathbb{Z}_{2^2} \oplus \mathbb{Z}_{5^2} \\
\mathbb{Z}_{240} &\simeq \mathbb{Z}_{2^4} \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_5
\end{aligned}$$

thus

$$B \simeq \mathbb{Z}_2 \oplus \mathbb{Z}_{2^2} \oplus \mathbb{Z}_{2^2} \oplus \mathbb{Z}_{2^4} \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_{5^2}$$

and $\{2, 4, 4, 16, 3, 3, 3, 5, 5, 25\}$ are the elementary divisors of B.

2	2^2	2^2	2^4
	3	3	3
	5	5	5^2
2	60	60	1200

which are the four invariant factors of B.