ECE393 – Lab 3

Jonathan Lam

December 23, 2020

1 Project summary

This lab follows Lab 2-4: "Low-pass filter" in the *Student Manual for the Art of Electronics*. This lab involves the analysis of a simple first-order RC LPF, such as its 3dB point and asymptotic behavior. This involves a theoretical derivation of the gain (magnitude and phase) characteristics, followed by a LTSpice simulation and a physical implementation to empirically demonstrate the physical results. The theory closely matches the experimental results.

2 Setup

We have the circuit in Figure 1, with $R = 15\text{k}\Omega$ and $C = 0.01\mu\text{F}$. Using

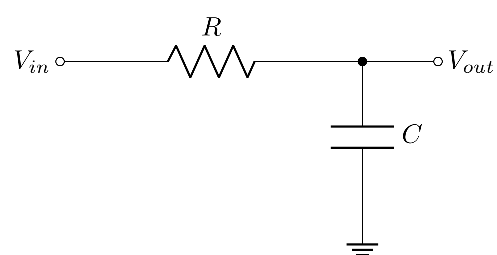


Figure 1: Generic RC LPF circuit

the frequency-domain representation of the circuit, we can make the capacitor a linear resistor with complex impedance $\frac{1}{j\omega C}$. Then, the circuit becomes a voltage divider, as shown in Figure 2. The gain of the voltage divider is the

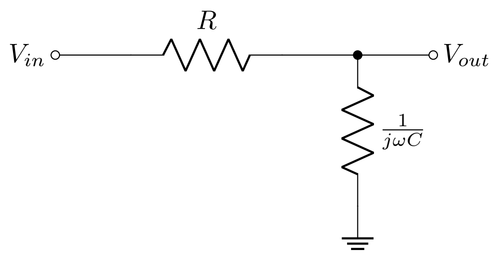


Figure 2: RC LPF becomes voltage divider with complex impedances

same as any voltage divider:

$$
\begin{aligned}
A_V &= \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} \\
|A_V| &= \left| \frac{1}{1 + j\omega RC} \right| = \frac{|1|}{|1 + j\omega RC|} = \frac{1}{\sqrt{1^2 - j^2\omega^2 R^2 C^2}} = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} \\
\angle A_V &= \angle(1) - \angle(1 + j\omega RC) = 0 - \arctan\left(\frac{\omega RC}{1}\right) = -\arctan \omega RC
\end{aligned}
$$

From these derivations, it is clear that the magnitude of the gain is upper limited at one (and this is its value as $\omega = 0$). We also note that the phase shift is between 0 (at $\omega = 0$) and $-\pi/2$ (as $\omega \to \infty$), which confirms our understanding that resistors do not alter phase and capacitors perform a negative 90-degree phase shift.

3 3dB point

The 3dB point is when the the power gain is $\frac{1}{2}$, or the voltage gain is $\frac{1}{\sqrt{2}}$. Thus:

$$
\begin{aligned}
|A_V| = \frac{1}{\sqrt{2}} &= \frac{1}{\sqrt{1 + \omega_{3\text{dB}}^2 R^2 C^2}} \\
&\Rightarrow 2 = 1 + \omega_{3\text{dB}}^2 R^2 C^2 \\
&\Rightarrow \omega_{3\text{dB}} = \frac{1}{RC}
\end{aligned}
$$

In this case, $\omega_{3\text{dB}} = ((15\text{k}\Omega)(0.01\mu\text{F}))^{-1} = 6.67 \times 10^3\text{rad/s}$, or $f_{3\text{dB}} = 1.06 \times 10^3\text{kHz}$. At this frequency, the phase shift is:

$$
\angle A_V = -\arctan\left(\frac{1}{RC} RC\right) = -\arctan 1 = -\frac{\pi}{4}
$$

4 6dB per octave

From the above derivation, we see that $\omega^2 R^2 C^2 = 1$ at the 3dB point. At frequencies (much) smaller than the 3dB point, then $\omega^2 R^2 C^2 \ll 1$ and $|A_V| \approx$

1; at frequencies (much) larger than the 3dB point, then $\omega^2 R^2 C^2 \gg 1$, and $|A_V| \approx (\omega RC)^{-1}$. In this case, doubling ω halves the gain (a 6dB attenuation). E.g., if we look at ten and twenty times the the 3dB point, we should see half the gain in the latter.

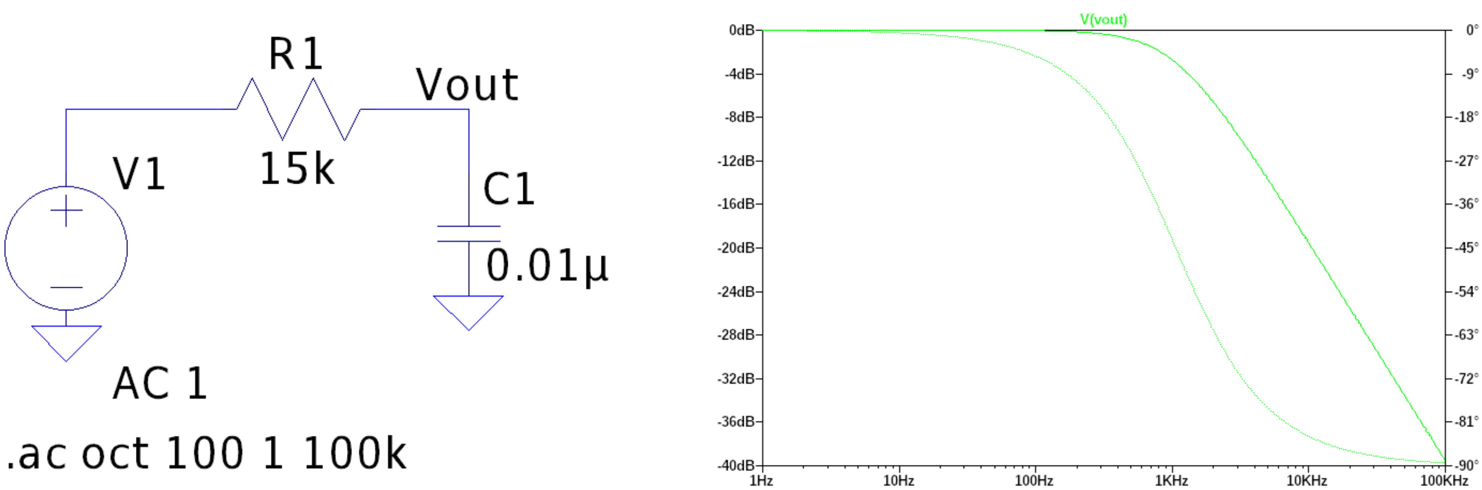
5 Integrator vs. LPF

Is the integrator a special case of a low-pass filter, or is the LPF a special case of the integrator?

The LPF is a specific case (i.e., specific application) of integrators. Using the geometric interpretation of integration as the (signed) area under a curve, a high-frequency signal has a very small amplitude under the wave because it quickly alternates, while a low-frequency signal fluctuates less slowly and can have a larger area under the curve before the signal changes sign again. Thus the integrator is functionally a LPF – it is not an ideal LPF (i.e., it has that logarithmic "roll-off" of 6dB per octave), but it attenuates higher frequences much more than lower frequencies.

6 LTSpice Simulations

We can simulate this circuit in LTSpice (Figure 3a) and perform an AC sweep on the gain (Figure 3b). The upper line in Figure 3b is the magnitude of the gain, and the lower line is the phase. As expected, the phase goes from 0 (when $\omega = 0$) to $-\pi/2$ radians (as ω becomes large).



(a) LPF schematic in LTSpice

(b) Voltage gain magnitude and phase plotted as a function of a (logarithmic) frequency sweep

Figure 3: LTSpice simulation

If we zoom in on the 3dB (more close to 3.01dB) point, we can see that it occurs at 1.06kHz and $-\pi/4$ phase shift, as previously calculated.

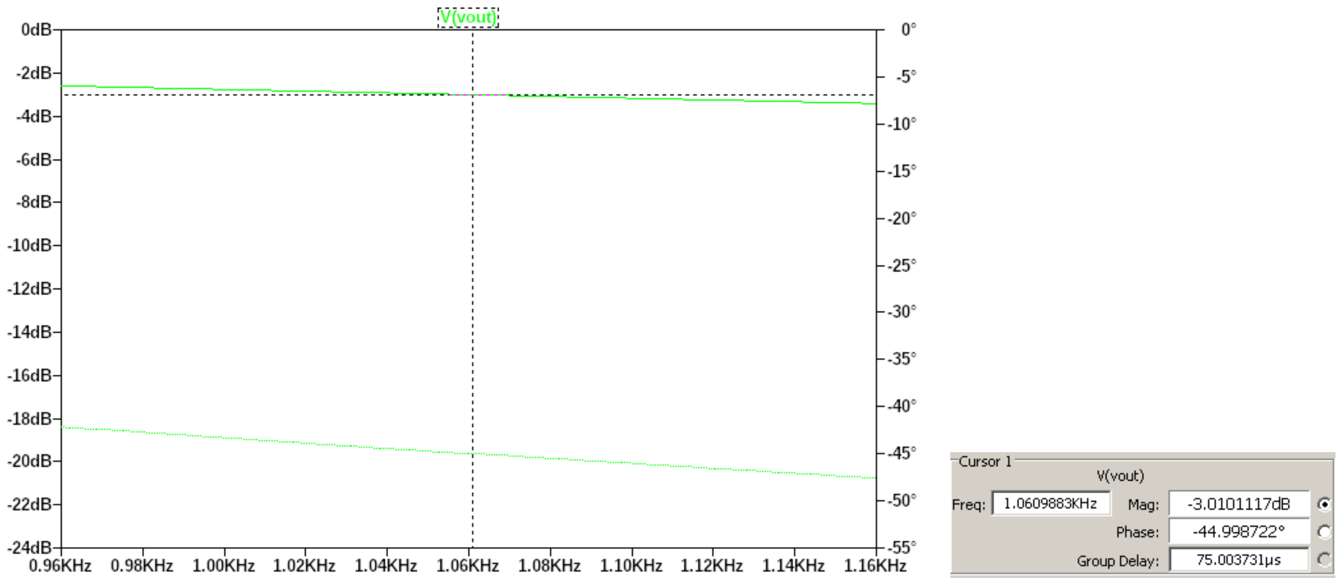
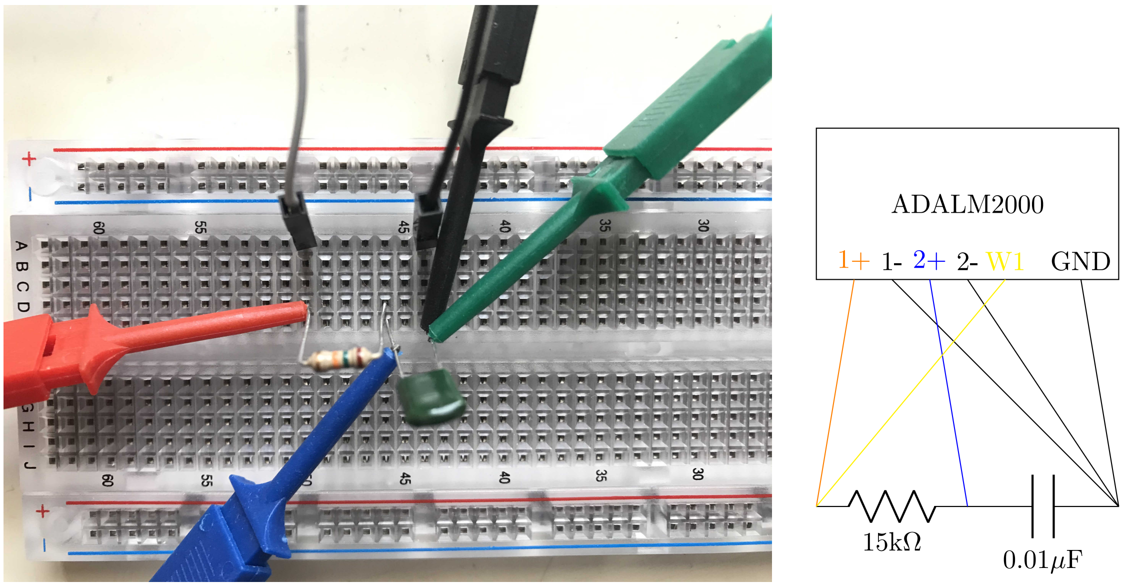


Figure 4: Close-up of 3dB point

7 Experimental results

The circuit was implemented on a breadboard and the signal generator and oscilloscope functions on the ADALM2000. The Scopy program was used to view the results on a computer.



(a) Photo of realization

(b) Block diagram showing connections with ADALM2000

Figure 5: Realization of circuit. The gray and black jumpers are connected to the waveform generator (ADALM W1 and GND). The red and black probes (across the signal generator, V_{in}) are for scope 1 (ADALM 1+, 1- pins); the blue and green probes (across the capacitor, V_{out}) are for scope 2 (ADALM 2+, 2- pins)).

The 3dB point was found experimentally by driving a 1V amplitude sinusoid at a frequency such that the amplitude of the voltage over the capacitor is roughly 0.707V (3dB). For this setup, this was roughly 1.00kHz, which is not far from the theoretical 1.06kHz – this is reasonable given the tolerance of the resistor, capacitor, and ADALM2000. The phase shift can be calculated by $2\pi\Delta t/T = 2\pi(-1.22 \times 10^{-4}\text{s})/(1.00 \times 10^{-3}\text{s}) = -0.244\pi$ (a negative phase shift indicates a delay, as we see in the figure), which is close to the theoretical $-\pi/4$.

When we plot this against a larger range of frequencies, as in Figure 7, the results match the theoretical results very closely. (The experimental and theoretical trends are largely indistinguishable for lower frequencies and diverge a little at the higher frequencies, but this divergence is likely due to instrumental error due to the nature of the high frequencies and the small amplitude of the

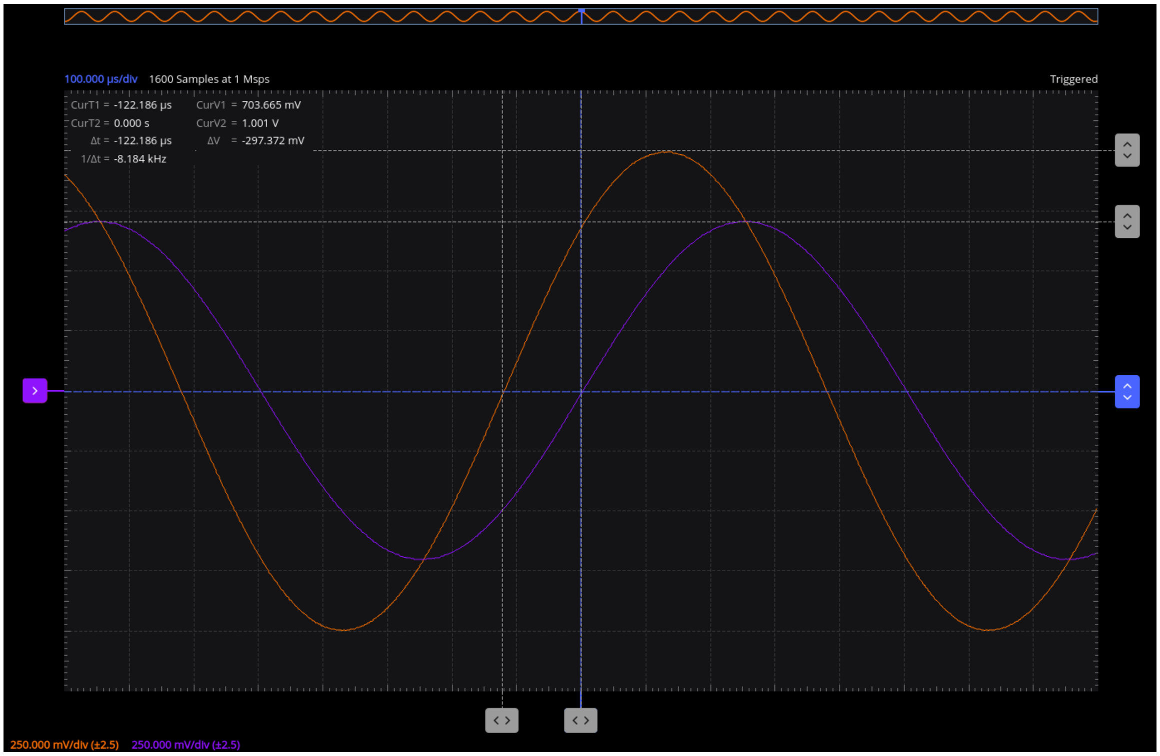
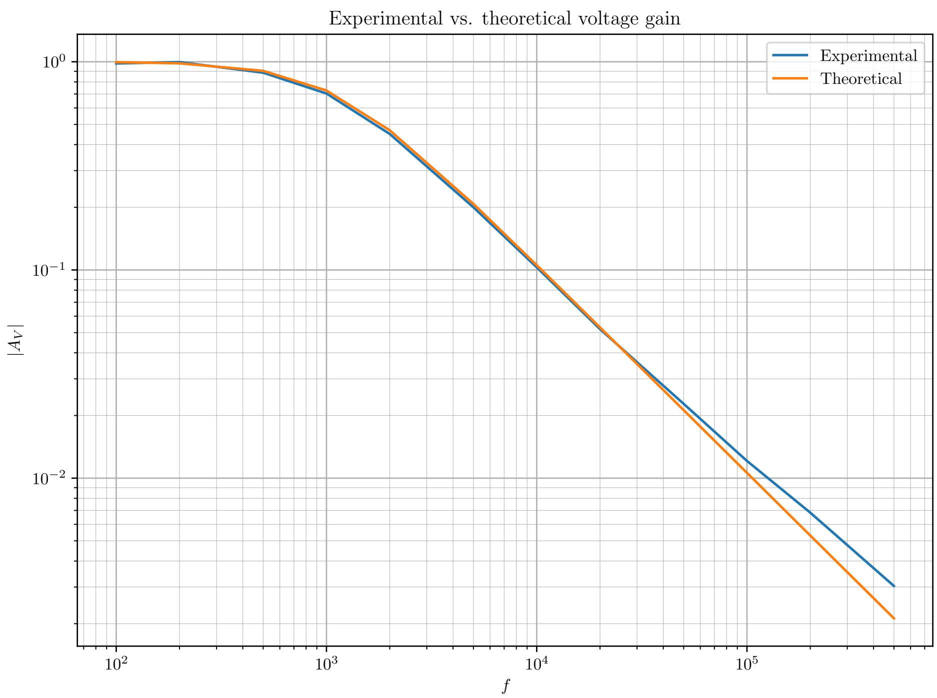


Figure 6: A plot of the experimentally-determined 3dB point on the Scopy oscilloscope. This is being driven by a 1.00kHz sine wave. The plot has 100μs/div. on the x-axis, 250mV/div. on the y-axis. The horizontal cursors are placed at $t = 0\mu$s and $t = -122\mu$s for the zero-crossing points of V_{out} and V_{in}, respectively ; the vertical cursors are placed at $V = 0.704$V and $V = 1.00$V, for the amplitudes of V_{out} and V_{in}, respectively.

attenuated signal.) We also see the asymptotic behavior as $\omega \to \infty$; that is, the 6dB/octave loss from 10.0kHz to 20.0kHz (or between 100.0kHz and 200.0kHz). This is equivalent to saying that doubling the frequency halves the gain; this is clear on the plot given the slope of negative one (on a log-log scale) for frequencies much higher than the 3dB frequency.



f (Hz)	$\lvert A_V \rvert$
100	0.982
200	0.997
500	0.886
1.00k	0.704
2.00k	0.450
5.00k	0.202
10.0k	0.103
20.0k	0.052
50.0k	0.0228
100k	0.0121
200k	0.00683
500k	0.00304

Figure 7: Experimental vs. theoretical gain results plotted on a log scale