ECE342 – Quiz 4

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1 MOS differential amplifier circuit

The circuit in Figure 1 uses transistors for which $\mu_n C_{ox} = 200\mu\text{A/V}^2$, $V_{TH} = 2\text{V}$ and $\lambda = 0.02\text{V}^{-1}$. The supplies are $V_{DD} = 12\text{V}$ and $V_{SS} = -12\text{V}$.

Resistor	Value (kΩ)
R_1	55
R_D	40
$R_{D,2}$	4
R_5	6

Table 1: Resistor values for question 1

1.1 DC analysis

The first thing to do is to choose a value of the aspect ratio W/L. Since this is not provided in the problem, I chose an arbitrary value that I believe is reasonable (similar to some of the values provided in the textbook problems):

$$\frac{W}{L} = 15 \tag{1}$$

Assume all transistors are in saturation mode – although it's not shown here, all of the transistors can be shown to be in saturation mode by showing that $V_{DS} > V_{GS} - V_{TH}$ and $V_{GS} > V_{TH}$. We first have to examine the biasing of the circuit, and thus the current mirror at M_4 and M_5. This can be solved for with iteration using the following two equations (see Figure 6 for the iteration source code):

$$V_{GS,4} = (V_{DD} - I_{D,4}R_1) - V_{SS} \tag{2}$$

$$I_{D,4} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS,4} - V_{TH})^2(1 + \lambda V_{DS,4}) \tag{3}$$

I tried these in Python but it wouldn't converge, so I tried rearranging them:

$$I_{D,4} = \frac{V_{DD} - V_{GS,4} - V_{SS}}{R_1} \tag{4}$$

$$V_{GS,4} = V_{TH} + \sqrt{\frac{2I_{D,4}L}{\mu_n C_{ox} W(1 + \lambda V_{DS,4})}} \tag{5}$$

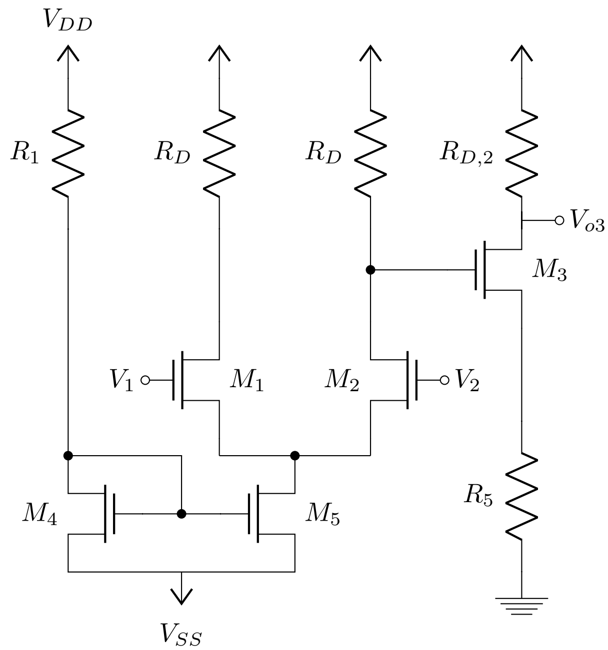

Figure 1: Schematic for question 1

Using the initial values $I_{D,1} = 1\text{mA}$ and $V_{GS,1} = 2.1\text{V}$ results in the values:

$$I_{D,4} = 0.391\text{mA} \tag{6}$$
$$V_{GS,4} = 2.50\text{V} \tag{7}$$

This current should be equal to the drain current through M_5, and equal to double the drain currents through the differential pair M_1 and M_2.

$$I_{D,4} = I_{D,5} = 2I_{D,1} = 2I_{D,2} \tag{8}$$
$$V_{D,1} = V_{D,2} = V_{DD} - I_{D,1}R_D = 12\text{V} - (0.196\text{mA})(40\text{k}\Omega) = 4.18\text{V} \tag{9}$$
$$V_{GS,1} = V_{GS,2} = V_{TH} + \sqrt{\frac{2I_{D,1}L}{\mu_n C_{ox} W(1 + \lambda V_{DS,1})}} \tag{10}$$

Assuming that initially the inputs are centered at DC 0V, then $V_{DS,1} = V_{D,1} - (0 - V_{GS,1}) = V_{D,1} + V_{GS,1}$. We can solve this by iterating this one equation until convergence (in Python):

$$V_{DS,1} = V_{DS,2} = 6.52\text{V} \tag{11}$$
$$V_{GS,1} = V_{GS,2} = 2.34\text{V} \tag{12}$$

Looking at the last transistor M_3, we know the gate voltage, but we do not know the source voltage, drain voltage, or drain current. However, these values can all be expressed as functions of $I_{D,3}$ and

$V_{GS,3}$, so we only need to solve for those two variables:

$$V_{G,3} = V_{D,2} \tag{13}$$
$$V_{D,3} = V_{DD} - I_{D,3}R_{D,2} \tag{14}$$
$$V_{S,3} = V_{G,3} - V_{GS,3} \tag{15}$$

We can solve for the two missing variables $I_{D,3}$ and $V_{GS,3}$ by iterating the two equations:

$$I_{D,3} = \frac{(V_{G,3} - V_{GS,3}) - V_{SS}}{R_5} \tag{16}$$
$$V_{GS,3} = V_{TH} + \sqrt{\frac{2I_{D,3}L}{\mu_n C_{ox} W(1 + \lambda((V_{DD} - I_{D,1}R_{D,2}) - (V_{G,3} - V_{GS,3})))}} \tag{17}$$

where the long last term in the denominator is equal to $V_{DS,3}$. By iterating in Python with initial values $I_{D,3} = 1\text{mA}$ and $V_{GS,3} = 2.1\text{V}$, we get:

$$I_{D,3} = 0.295\text{mA} \tag{18}$$
$$V_{GS,3} = 2.41\text{V} \tag{19}$$

and from this we are able to calculate the rest of the voltages:

$$V_{D,3} = V_{DD} - I_{D,3}R_{D,2} = 12\text{V} - (0.295\text{mA})(4\text{k}\Omega) = 10.8\text{V} \tag{20}$$
$$V_{S,3} = V_{G,3} - V_{GS,3} = 4.18\text{V} - 2.41\text{V} = 1.78\text{V} \tag{21}$$

1.2 Differential mode voltage gain

Decompose the problem into two stages: the first stage is the differential pair with M_1 and M_2 with gain $A_{V,1}$ and the second stage is the CS stage M_3 with voltage gain $A_{V,2}$. The total gain is the product of these gains.

For the differential pair, the voltage gain is equal to half of the voltage gain of one of the CS transistors. It is only half because only one of the differential pair outputs is used.

$$A_{V,1} = \frac{1}{2}(-g_{m,1}(R_D||R_{O,1})) \tag{22}$$
$$R_{O,1} \approx \frac{1}{\lambda I_{D,1}} = \frac{1}{(0.02\text{V}^{-1})(0.391/2\text{mA})} = 256\text{k}\Omega \tag{23}$$
$$g_{m,1} = \mu_n C_{ox}\frac{W}{L}(V_{GS,1} - V_{TH}) = (200\mu\text{A}/\text{V}^2)(15)(2.34\text{V} - 2\text{V}) = 0.00102\Omega^{-1} \tag{24}$$
$$A_{V,1} = -\frac{(0.00102\Omega^{-1})(40\text{k}\Omega||256\text{k}\Omega)}{2} = -17.6 \tag{25}$$

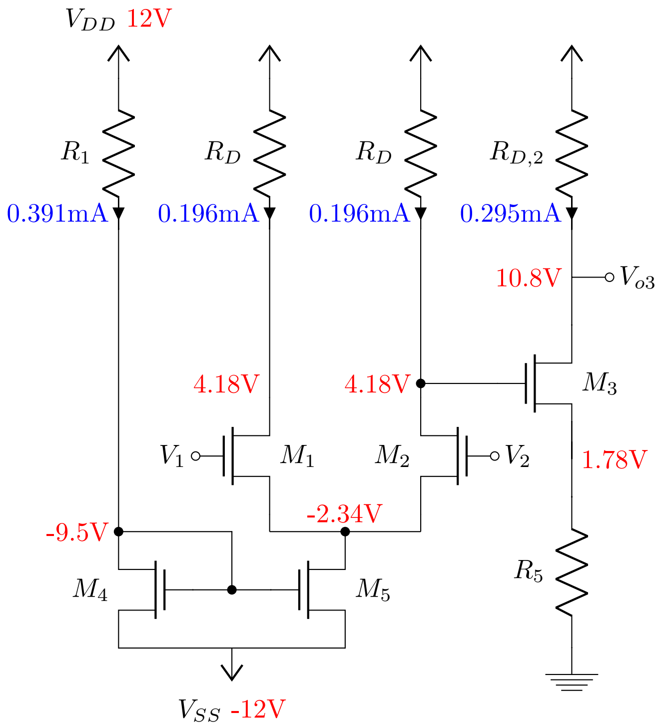

Figure 2: Schematic for question 1 labeled with DC voltages and currents

The second stage is a CS stage with emitter degeneration:

$$A_{V,2} = \frac{-R_{D,2}||R_{O,3}}{\frac{1}{g_{m,3}} + R_{S,3}} \tag{26}$$

$$R_{O,3} \approx \frac{1}{\lambda I_{D,3}} = \frac{1}{(0.02\text{V}^{-1})(0.295\text{mA})} = 169\text{k}\Omega \tag{27}$$

$$g_{m,3} = \mu_n C_{ox}(V_{GS,3} - V_{TH}) = (200\mu\text{A}/\text{V}^2)(15)(2.41\text{V} - 2\text{V}) = 0.00123\Omega^{-1} \tag{28}$$

$$A_{V,2} = -\frac{4\text{k}\Omega||169\text{k}\Omega}{\frac{1}{0.00123\Omega^{-1}} + 6\text{k}\Omega} = -0.574 \tag{29}$$

The total voltage gain is thus:

$$A_V = (-17.6)(-0.574) = 10.1 \tag{30}$$

1.3 Common mode voltage gain

The common mode voltage gain of a differential pair is zero due to its CM rejection property. Thus, the first stage would have a CM voltage gain of zero, leading to the entire circuit having a CM voltage gain of zero.

2 Bipolar differential amplifier circuit

Consider the bipolar op-amp circuit in Figure 3. For simplicity, you may assume $|V_{BE}| = 0.7\text{V}$, $\beta = 100$, and $V_A = \infty$. The supplies are $V_{DD} = 15\text{V}$ and $V_{SS} = -15\text{V}$.

Note: I rename the resistors for sake of my sanity. Each resistor is labeled with C or E and a number to indicate what transistor it is associated with, and whether it is connected the collector or the emitter of that transistor.

Resistor	Value (kΩ)
$R_{C,1}$	20
$R_{C,2}$	20
$R_{C,5}$	3
$R_{E,7}$	2.3
$R_{C,7}$	15.7
$R_{E,8}$	3
$R_{C,9}$	28.6

Table 2: Resistor values for question 2

2.1 DC analysis

Like in the previous question, assume all BJTs are in FA mode. This is not checked explicitly, but can be checked by making sure that $V_{CE} > V_{BE}$. We start by examining the current mirror at Q_9:

$$V_{B,9} = V_{EE} + V_{BE} = -15\text{V} + 0.7\text{V} = -14.3\text{V} = V_{C,9} \tag{31}$$

$$I_{C,9} = \frac{\text{GND} - V_{C,9}}{R_{C,9}} = \frac{0 - (-14.3\text{V})}{28.6\text{k}\Omega} = 0.5\text{mA} \tag{32}$$

Since this current mirror's current is only being replicated $n = 5$ times and β is fairly large, we can approximate the copied current to be equal to the reference one. I.e.:

$$I_{C,3} = I_{C,9} = \frac{1}{4} I_{C,6} \tag{33}$$

Now we have the emitter current biases for the two differential pairs. Since all of the V_{BE}s are equal, we would expect the currents $I_{C,1}$ and $I_{C,2}$ to be equal and half of $I_{C,3}$. Neglecting any base currents:

$$I_{C,1} \approx I_{C,2} \approx I_{E,1} = I_{E,2} = \frac{1}{2} I_{C,3} = 0.25\text{mA} \tag{34}$$

$$V_{C,1} \approx V_{C,2} \approx V_{CC} - I_{C,1} R_{C,1} = 15\text{V} - (0.25\text{mA})(20\text{k}\Omega) = 10\text{V} \tag{35}$$

We also assume the inputs would be centered at DC 0V, thus:

$$V_{B,1} = V_{B,2} = 0\text{V} \tag{36}$$

$$V_{E,1} = V_{E,2} = V_{C,3} = V_{B,1} - V_{BE} = 0\text{V} - 0.7\text{V} = -0.7\text{V} \tag{37}$$

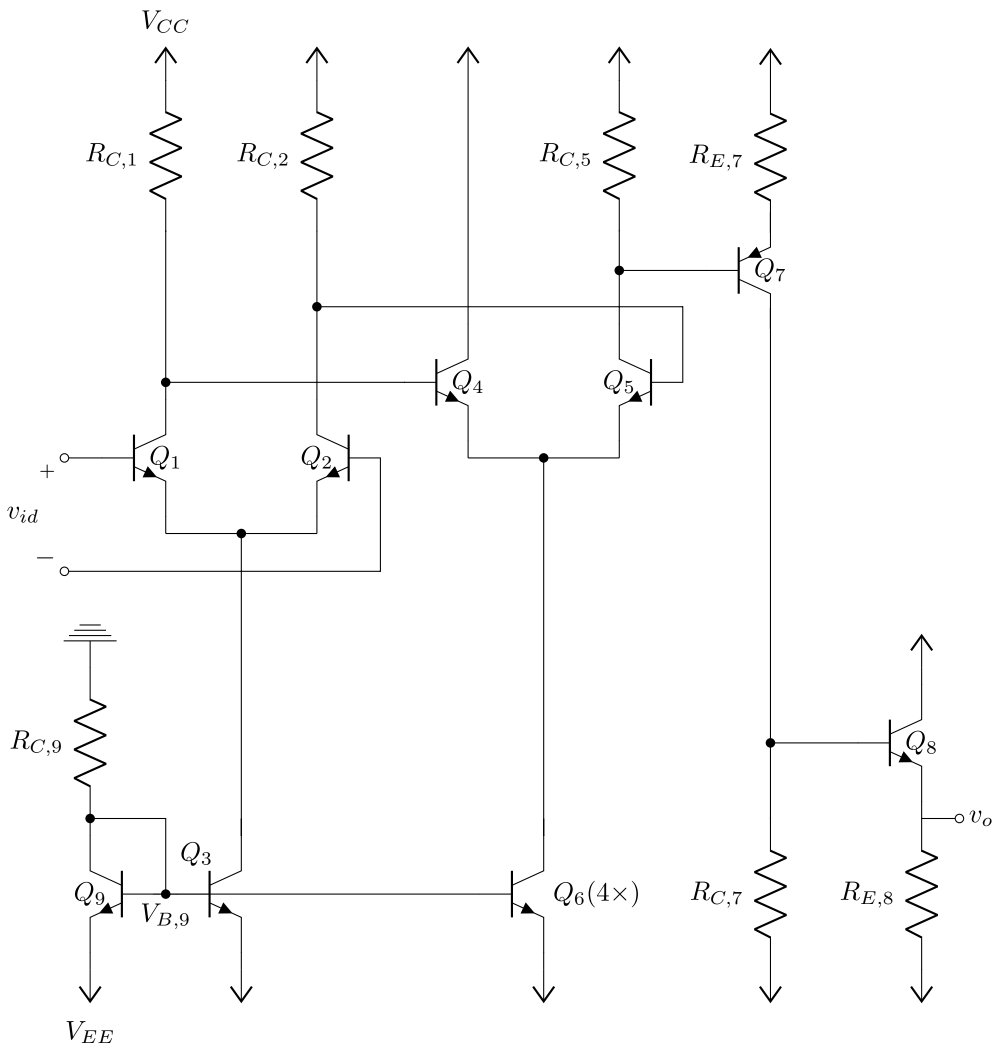

Figure 3: Schematic for question 2

Moving onto the second differential pair, again assume that the currents are split evenly between Q_4 and Q_5:

$$I_{C,4} \approx I_{C,5} \approx I_{E,4} = I_{E,5} = \frac{1}{2} I_{Q,6} = \frac{1}{2}(4)(0.5\text{mA}) = 1\text{mA} \tag{38}$$

$$V_{C,5} = V_{CC} - I_{C,5}R_{C,5} = 15\text{V} - (1\text{mA})(3\text{k}\Omega) = 12\text{V} \tag{39}$$

$$V_{B,4} = V_{B,5} = V_{C,1} = V_{C,2} = 10\text{V} \tag{40}$$

$$V_{E,4} = V_{E,5} = VC,6 = V_{B,4} - V_{BE} = 10\text{V} - 0.7\text{V} = 9.3\text{V} \tag{41}$$

Looking at Q_7 and Q_8:

$$V_{B,7} = V_{C,5} = 12\text{V} \tag{42}$$

$$V_{E,7} = V_{B,7} + V_{EB} = 12\text{V} + 0.7\text{V} = 12.7\text{V} \tag{43}$$

$$I_{E,7} = \frac{V_{CC} - V_{E,7}}{R_{C,7}} = \frac{15\text{V} = 12.7\text{V}}{2.3\text{k}\Omega} = 1\text{mA} \approx I_{C,7} \tag{44}$$

$$V_{C,7} = V_{EE} + I_{C,7}R_{C,7} = -15\text{V} + (1\text{mA})(15.7\text{k}\Omega) = 0.7\text{V} = V_{B,8} \tag{45}$$

$$V_{E,8} = V_{B,8} - V_{BE} = 0.7\text{V} - 0.7\text{V} = 0V = v_o \tag{46}$$

$$I_{E,8} = \frac{V_{E,8} - V_{EE}}{R_{E,8}} = \frac{0\text{V} - (-15\text{V})}{3\text{k}\Omega} = 5\text{mA} \approx I_{C,8} \tag{47}$$

2.2 Quiescent power dissipation

To find this, we look at the sum of the DC currents at the voltage sources multiplied by the source voltages. We then take the absolute value since power is always positive.

$$\begin{aligned} P &= \sum |IV| \\ &= |((2)(0.25\text{mA}) + (2)(1\text{mA}) + 1\text{mA} + 5\text{mA})(15\text{V})| \\ &\quad + |(0.5\text{mA} + 0.5\text{mA} + 2\text{mA} + 1\text{mA} + 5\text{mA})(-15\text{V})| \\ &= 262.5\text{mW} \end{aligned} \tag{48}$$

2.3 Voltage gain

We break this up into stages. Call the first differential pair (Q_1, Q_2) the first stage with gain $A_{V,1}$; the second differential pair (Q_4, Q_5) the second stage with gain $A_{V,2}$; the PNP common-emitter Q_7 the third stage with gain $A_{V,3}$, and the emitter follower Q_8 the final stage with gain $A_{V,4}$. (Note that this deviates from the earlier convention where the index denotes the associated transistor; here the index denotes the stage for which we're examining the voltage gain) The total gain is the product of these values, i.e,:

$$A_V = \prod_{i=1}^{4} A_{V,i} \tag{49}$$

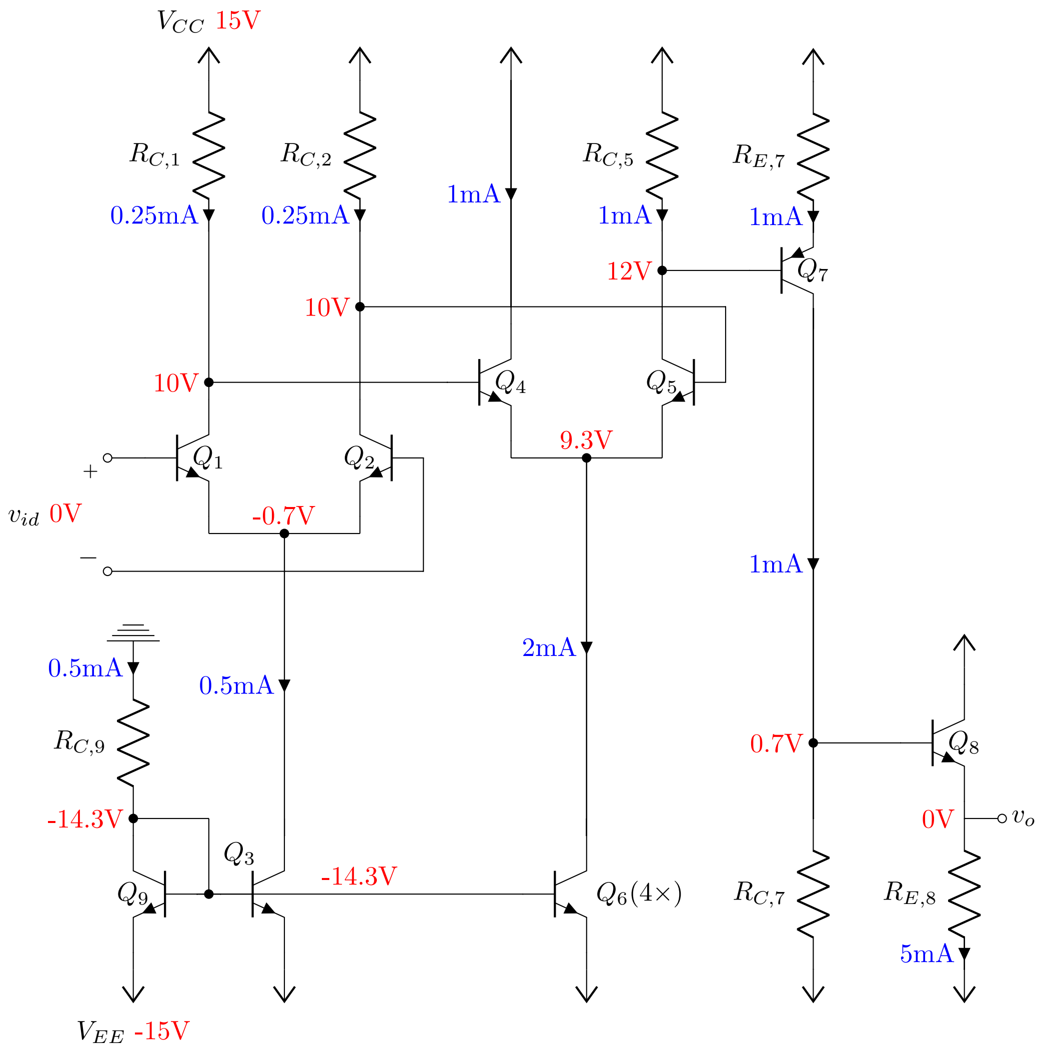

Figure 4: Schematic for question 2 with labeled DC currents and voltages

Looking at the first differential pair, Q_1 and Q_2 are symmetric and the differential voltage gain is equal to the voltage gain of either one.

$$A_{V,1} = -g_{m_1} R_{out,1} \tag{50}$$
$$g_{m_1} = \frac{I_C}{V_T} = \frac{0.25\text{mA}}{26\text{mV}} = 0.00962\Omega^{-1} \tag{51}$$
$$R_{out,1} = R_{C,1} || R_{in,4} \tag{52}$$

Due to the "virtual ground" nature of the emitter in a differential pair, it is easy to find the input resistance of each common emitter in a differential pair:

$$R_{in,4} = R_{\pi,4} = \frac{\beta}{g_m} = \frac{100}{\frac{1\text{mA}}{26\text{mV}}} = 2600\Omega \tag{53}$$
$$A_{V,1} = -g_{m1} R_{out,1} = -(0.00962\Omega^{-1})(20\text{k}\Omega||2.6\text{k}\Omega) = -22.1 \tag{54}$$

Looking at the second differential pair, the output only uses one end of the differential pair, so the voltage gain is half of the voltage gain of the Q_5 CE:

$$A_{V,2} = \frac{1}{2} g_{m,5} R_{out,5} \tag{55}$$
$$g_{m,5} = \frac{I_{C,5}}{V_T} = \frac{1\text{mA}}{26\text{mV}} = 0.0385\Omega^{-1} \tag{56}$$
$$R_{out,5} = R_{C,5} || R_{in,7} \tag{57}$$

Q_7 is just a single CE transistor (with emitter degeneration), so its input impedance is given by:

$$R_{in,7} = R_{\pi.7} + R_{E,7}(\beta+1) = \frac{\beta}{g_{m,7}} + R_{E,7}(\beta+1) = \frac{100}{\frac{1\text{mA}}{26\text{mV}}} + (2.3\text{k}\Omega)(101) = 235\text{k}\Omega \tag{58}$$
$$A_{V,2} = -\frac{1}{2} g_{m,5}(R_{C,5}||R_{in,7}) = -\frac{1}{2}(0.00385\Omega^{-1})(3\text{k}\Omega||235\text{k}\Omega) = -57.0 \tag{59}$$

Now, looking at the third stage, Q_7, which is a CE stage with degeneration:

$$A_{V,3} = -\frac{g_{m,7} R_{out,7}}{1 + g_{m,7} R_{E,7}} \tag{60}$$
$$g_{m,7} = \frac{I_{C,7}}{V_T} = \frac{1\text{mA}}{26\text{mV}} = 0.0385\Omega^{-1} \tag{61}$$
$$R_{out,7} = R_{C,7} || R_{in,8} \tag{62}$$
$$R_{in,8} = R_{\pi,8} + R_{E,8}(\beta+1) = \frac{\beta}{g_{m,8}} + R_{E,8}(\beta+1) = \frac{100}{\frac{5\text{mA}}{26\text{mV}}} + 3.0\text{k}\Omega(101) = 304\text{k}\Omega \tag{63}$$
$$A_{V,3} = -\frac{g_{m,7}(R_{C,7}||R_{in,8})}{1 + g_{m,7} R_{E,7}} = -\frac{(0.0385\Omega^{-1})(15.7\text{k}\Omega||304\text{k}\Omega)}{1 + (0.0385\Omega^{-1})(2.3\text{k}\Omega)} = -6.42 \tag{64}$$

And finally, the last stage, Q_8, which is an emitter follower:

$$A_{V,4} = \frac{R_{E,8}}{R_{E,8} + \frac{1}{g_{m,8}}} = \frac{3\text{k}\Omega}{3\text{k}\Omega + \frac{26\text{mV}}{5\text{mA}}} = 0.998 \tag{65}$$

Thus the total gain is:

$$A_V = \prod_{i=1}^{4} A_{V,i} = (-22.1)(-57.0)(-6.42)(0.998) = -8070 \tag{66}$$

(There may be a lot of roundoff error here, due to rounding in between steps.)

2.4 Input and output impedances

2.4.1 Input impedance

Since we have a differential input (between two input terminals, not between an input terminal and ground), the setup is a little different, as shown in Figure 5.

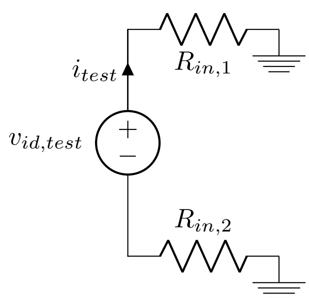

Figure 5: Input impedance setup for a two-terminal (differential) circuit

Due to the symmetry of Q_1 and Q_2, we expect $R_{in,1} = R_{in,2}$, so we expect that:

$$R_{in} = \frac{1}{2}(R_{in,2} + R_{in,1}) = R_{in,1} \tag{67}$$

Here, $R_{in,1}$ is a CE stage with degeneration, where the emitter impedance is the impedance looking into the emitter of Q_2 in parallel with the impedance looking into the collector of Q_3:

$$R_{in} = R_{in,1} = R_{\pi,1} + R_{E,1}(\beta + 1) \tag{68}$$

$$R_{E,1} = R_{\text{into emitter of } Q_2} || R_{\text{into collector of } Q_3} \tag{69}$$

$$R_{\text{into emitter of } Q_2} = \frac{R_{\pi,2}}{\beta + 1} \tag{70}$$

$$R_{\text{into collector of } Q_3} \approx \infty \tag{71}$$

$$R_{E,1} = \frac{R_{\pi,2}}{\beta + 1} || \infty = \frac{R_{\pi,2}}{\beta + 1} \tag{72}$$

$$R_{in} = R_{\pi,1} + \frac{\beta + 1}{\beta + 1} R_{\pi,2} = 2R_{\pi,1} \tag{73}$$

$$R_{\pi,1} = \frac{\beta}{\frac{I_{C,1}}{V_T}} = \frac{100}{\frac{0.25\text{mA}}{26\text{mV}}} = 10.4\text{k}\Omega \tag{74}$$

$$R_{in} = (2)(10.4\text{k}\Omega) = 20.8\text{k}\Omega \tag{75}$$

2.4.2 Output impedance

The output impedance is that of a emitter-follower with a resistance in series with the base (R_S in series with $R_{\pi,8}$). (This output impedance is Eq. 5.329 in the textbook.)

$$R_{in,8} = R_{E,8}|| \left(\frac{R_{out,7}}{\beta + 1} + \frac{1}{g_{m,8}} \right) \tag{76}$$

$$R_{out,7} = R_{C,7}||R_{\text{into collector of } Q_7} \tag{77}$$

$$R_{\text{into collector of } Q_7} \approx \infty \tag{78}$$

$$R_{out,7} = R_{C,7}||\infty = R_{C,7} \tag{79}$$

$$R_{in,8} = R_{E,8}|| \left(\frac{R_{C,7}}{\beta + 1} + \frac{1}{g_{m,8}} \right) = (3\text{k}\Omega)|| \left(\frac{15.7\text{k}\Omega}{101} + \frac{26\text{mV}}{5\text{mA}} \right) = 152\Omega \tag{80}$$

```python
import numpy as np
from math import sqrt

mu_c = 200e-6   # mu_n * C_{ox}
wol = 15        # W/L
v_th = 2        # V_{TH}
lam = 0.02      # lambda

r_1 = 55e3
r_d = 40e3
r_d2 = 4e3
r_5 = 6e3

v_dd = 12
v_ss = -12

# finding i_d4, v_gs4
i_d4 = 1e-3     # I_{D,4}
v_gs4 = 2.1     # V_{GS,4} = V_{DS,4}

def iterate():
    global i_d4, v_gs4
    i_d4 = (v_dd - v_gs4 - v_ss) / r_1
    v_gs4 = v_th + sqrt(2*i_d4/(wol * mu_c * (1 + lam * v_gs4)))
    print(f'i_d4 {i_d4} v_gs4 {v_gs4}')

print(f'i_d4 {i_d4} v_gs4 {v_gs4}')
for _ in range(5):
    iterate()

# finding v_gs1, v_gs2
i_d1 = i_d4/2
v_d1 = 12-r_d*i_d1
v_gs1 = 2.1

def iterate():
    global v_gs1
    v_ds1 = v_d1 + v_gs1
    v_gs1 = v_th + sqrt(2 * i_d1 / (mu_c * wol * (1 + lam * v_ds1)))
    print(f'v_ds1 {v_ds1} v_gs1 {v_gs1}')

print(f'v_gs1 {v_gs1}')
for _ in range(5):
    iterate()

# finding i_d3, v_gs3
v_g3 = v_d1
i_d3 = 1e-3
v_gs3 = 2.1

def iterate():
    global i_d3, v_gs3
    i_d3 = (v_g3 - v_gs3) / r_5
    v_d3 = v_dd - i_d3 * r_d2
    v_s3 = v_g3 - v_gs3
    v_ds3 = v_d3 - v_s3
    v_gs3 = v_th + sqrt(2 * i_d3 / (mu_c * wol * (1 + lam * v_ds3)))
    print(f'i_d3 {i_d3} v_gs3 {v_gs3}')

print(f'i_d3 {i_d3} v_gs3 {v_gs3}')
for _ in range(5):
    iterate()
```

Figure 6: Source code for iteration steps in question 1