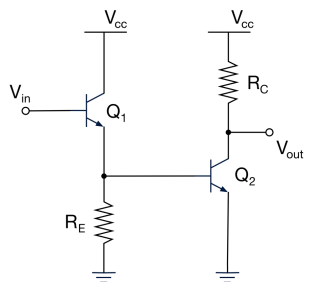
Name:

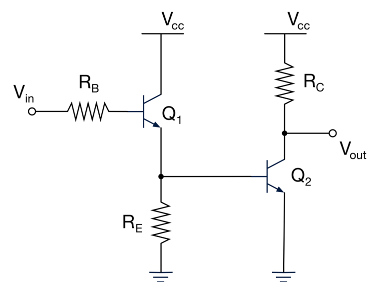
ECE 342-Electronics II
Quiz 2- Fall 2020

Problem 1

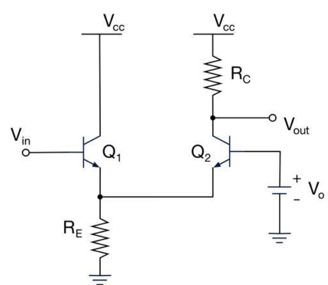
Find the gain for the following cascaded stage topologies and draw the small signal models. (Assume $V_A = \infty$)



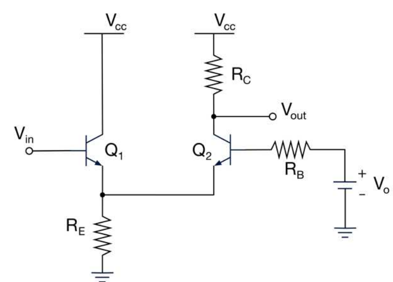
(a)



(b)



(c)

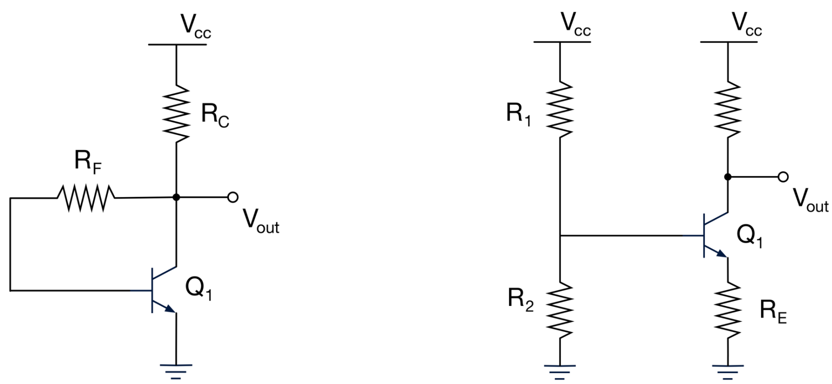


(d)

Problem 2

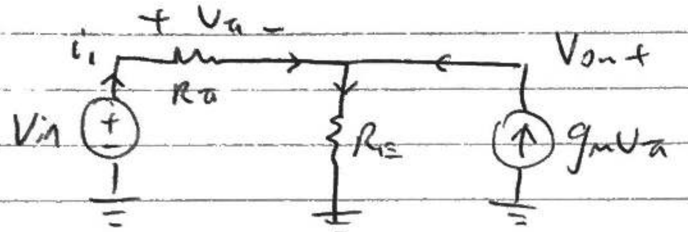
Consider the two C-E stages below, where Vcc = 2.5 V, $\beta = 100$, and $I_s = 5\text{x}10^{-16}$A. Perform the following:

a. Bias the circuits such that $I_c = 1$mA
b. How much will V_{BE} change if V_{CC} is increased by 5%? Which circuit is less sensitive to variations in V_{CC} for the specific values that you chose for biasing the circuits.
c. Explain the mechanism by which each circuit inhibits changes in V_{BE} due to variation in V_{CC}.



(a) (b)

EF



Finding A_v for emitter follower

1

$$i_1 + g_m v_\pi = \frac{V_{out}}{R_E}, \quad v_\pi = V_{in} - V_{out}, \quad i_1 = \frac{V_\pi}{R_\pi}$$

$$\frac{v_\pi}{R_\pi} + g_m v_\pi = \frac{V_{out}}{R_E}$$

$$\frac{V_{in} - V_{out}}{R_\pi} + g_m (V_{in} - V_{out}) = \frac{V_{out}}{R_E}$$

$$V_{in}\left(\frac{1}{R_\pi}\right) + V_{in}(g_m) = \frac{V_{out}}{R_E} + V_{out}\left(\frac{1}{R_\pi}\right) + V_{out}(g_m)$$

$$V_{in}\left(\frac{1}{R_\pi} + g_m\right) = V_{out}\left(\frac{1}{R_E} + \frac{1}{R_\pi} + g_m\right)$$

$$g_m = \frac{\beta}{R_\pi}$$

$$V_{in}\left(\frac{\beta+1}{R_\pi}\right) = V_{out}\left(\frac{\beta+1}{R_\pi} + \frac{1}{R_E}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{\beta+1}{R_\pi}}{\frac{\beta+1}{R_\pi} + \frac{1}{R_E}} = \frac{R_E}{R_E + \frac{R_\pi}{\beta+1}}$$

$$\frac{r_\pi}{\beta+1} \approx \frac{1}{g_m} \qquad = \frac{R_E}{R_E + \frac{1}{g_m}}$$

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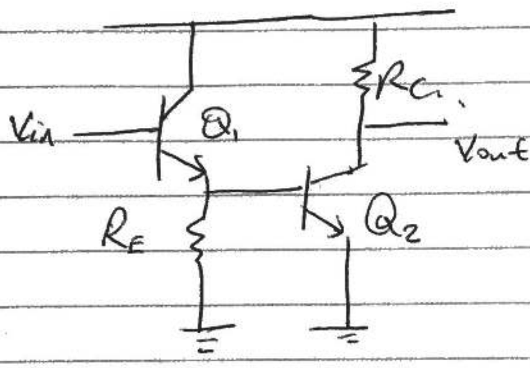
when there is an ~~~~ resistor R_B at the base, it is in series with r_π, it would make it like:

$$A_v = \frac{R_E}{R_E + \frac{R_\pi + R_B}{\beta+1}} \approx \frac{R_E}{R_E + \frac{1}{g_m} + \frac{R_B}{\beta+1}}.$$

ECE342 Quiz II

1.) Find the gain:



First stage

A_{v_1} of emitter follower stage $= \dfrac{R_E'}{R_E' + \frac{1}{g_m}}$.

where $R_E' = R_E \parallel R_{in_2}$.

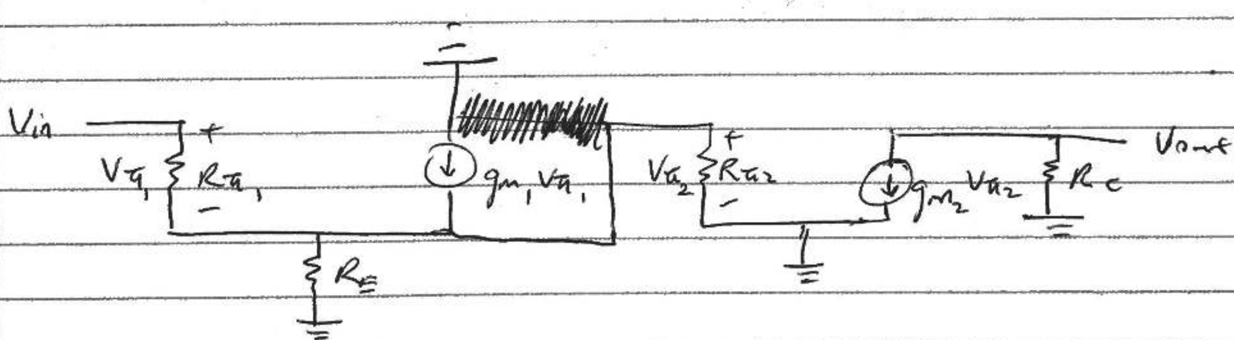
$R_{in_2} = R_{\pi_2}$ (input impedance of a CE stage).

Second stage

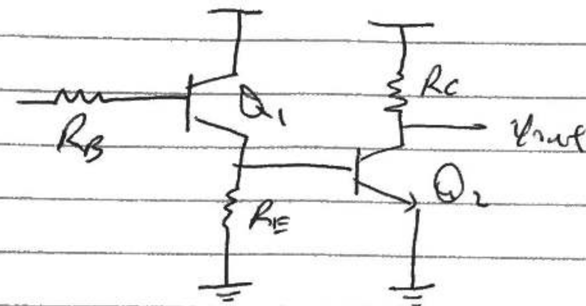
$A_{v_2} = -g_{m_2} R_C$

total gain $= A_{v_1} \cdot A_{v_2} = \dfrac{R_E \parallel R_{\pi_2}}{(R_E \parallel R_{\pi_2}) + \frac{1}{g_{m_1}}} \left(-g_{m_2} R_C\right)$.

Small signal model.



b)



First stage

$$A_{v_1} = \frac{R_E'}{R_E' + \frac{1}{g_{m_1}} + \frac{R_B}{\beta_1+1}}$$

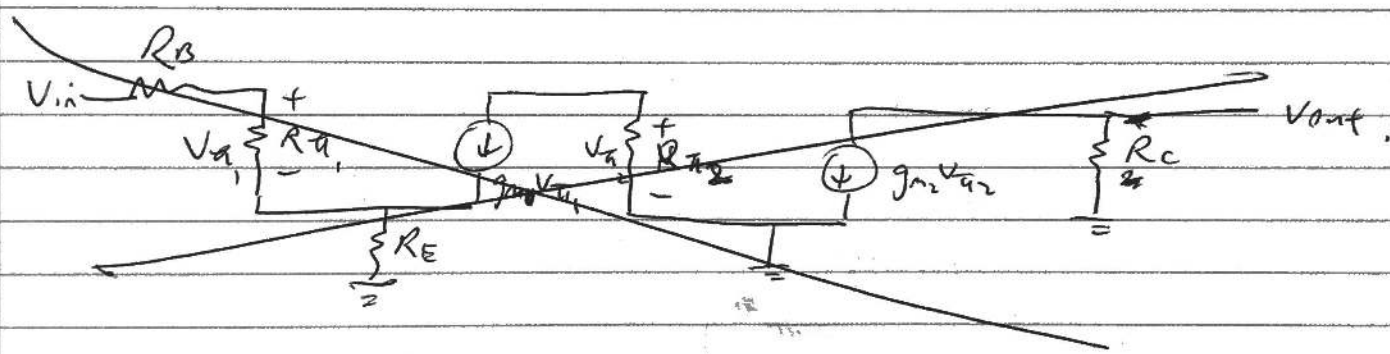
where $R_E' = R_E \,\|\, R_{in_2}$.

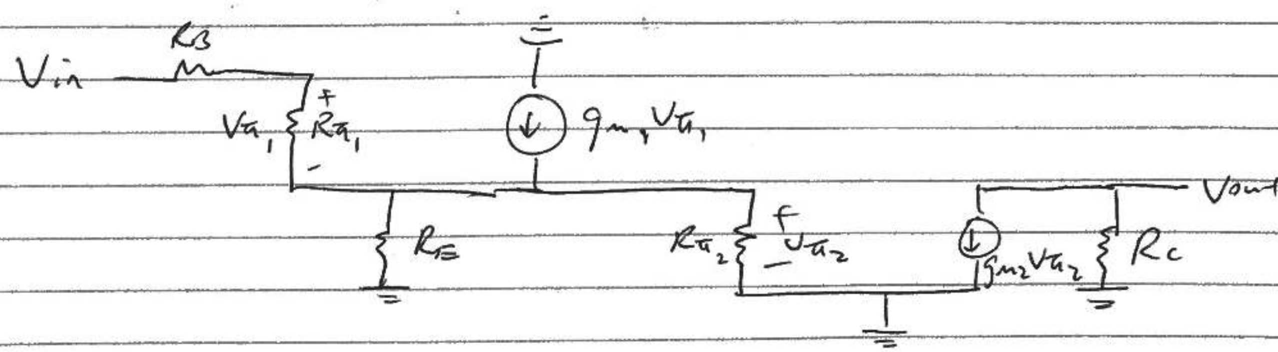
$R_{in\,2} = R_{\pi_2}$.

$$A_{v_2} = -g_{m_2} R_C.$$

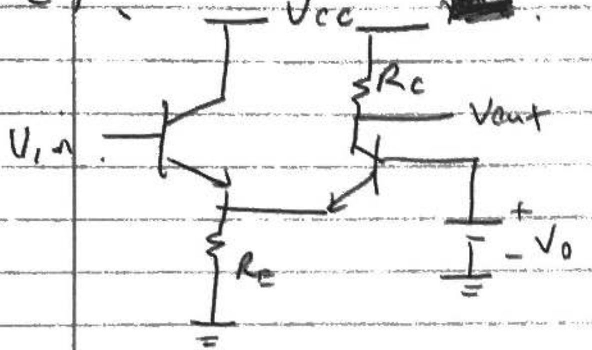
total gain: $A_v = A_{v_1} \cdot A_{v_2} = \left(\frac{R_E \,\|\, R_C}{(R_E \,\|\, R_C) + \frac{1}{g_{m_1}} + \frac{R_B}{\beta_1+1}} \right) \left(-g_{m_2} R_C\right)$

small-signal model:





c)



First stage

$$A_{v_1} = \frac{R_E'}{R_E' + \frac{1}{g_{m_1}}}$$

where $R_E' = R_E \parallel R_{in_2}$.

R_{in_2} is looking into the emitter,

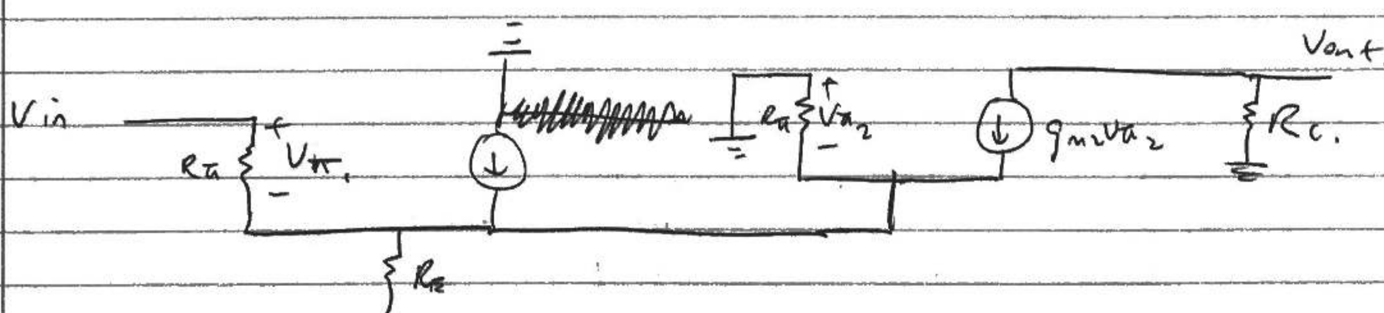
$$= \frac{r_{\pi_2}}{\beta_2 + 1} \approx \frac{1}{g_{m_2}}.$$

Second stage

$A_{v_2} = g_{m_2} R_C$ (common base)

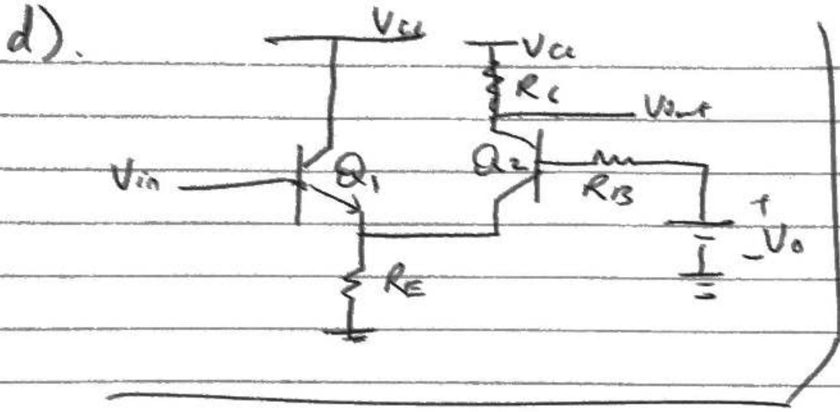
Total gain: $A_v = A_{v_1} \cdot A_{v_2} = \left(\dfrac{R_E \parallel \frac{1}{g_{m_2}}}{\left(R_E \parallel \frac{1}{g_{m_2}}\right) + \frac{1}{g_{m_1}}} \right) \left(g_{m_2} R_C \right)$

Small-signal model:



d).

(5)



Stage 1:

$$A_{v_1} = \frac{R_E}{R_E' + \frac{1}{g_{m_1}}}$$

$$R_E' = R_E \parallel R_{in_2}.$$

$$R_{in_2} = \frac{R_{\pi_2} + R_B}{\beta_2 + 1}$$

$$\approx \frac{1}{g_{m_2}} + \frac{R_B}{\beta_2 + 1}.$$

(b/c R_π in series w/ R_B).

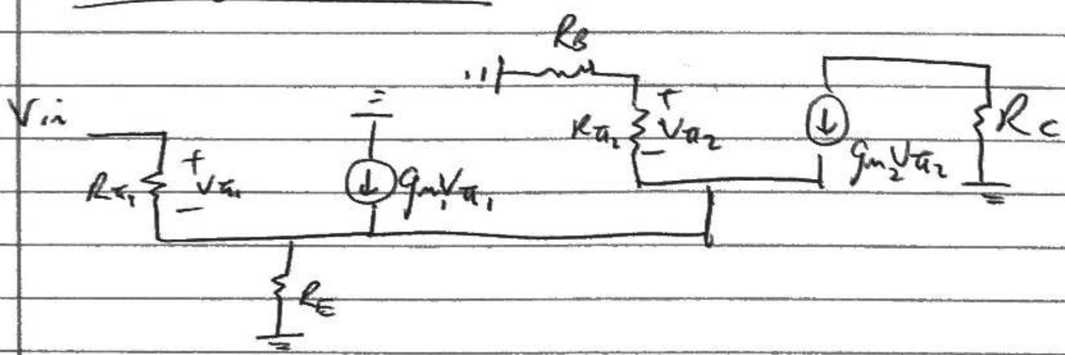
Second stage:

$$A_{v_2} = g_{m_2} R_C$$

Total gain:

$$A_v = A_{v_1} \cdot A_{v_2} = \left(\frac{R_E \parallel \left(\frac{1}{g_{m2}} + \frac{R_B}{\beta+1}\right)}{R_E \parallel \left(\frac{1}{g_{m2}} + \frac{R_B}{\beta+1}\right) + \frac{1}{g_{m_1}}} \right) \left(g_{m_2} R_C\right).$$

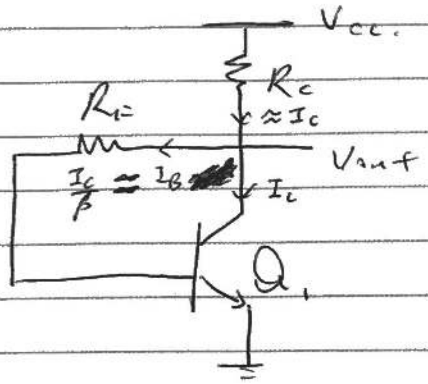
Small signal model:



Problem 2.

$V_{CC} = 2.5V,\ \beta = 100,\ I_S = 5\times10^{-16} A.$

a) Bias the circuits s.t. $I_C = 1mA$.



Use KVL:

$$V_{BE} + I_B R_F + I_C R_C = V_{CC}.$$

$$I_B = \frac{1}{\beta} I_C$$

$$V_{BE} + \frac{1}{\beta} I_C R_F + I_C R_C = V_{CC}.$$

We ~~[illegible]~~ want to make the voltage V_{CB} small so that we can approximate $\frac{V_{CC} - V_{out}}{R_C} = I_C$. Thus: $\frac{1}{\beta}\cancel{I_C} R_F \ll \cancel{I_C} R_C$

or $\frac{R_F}{\beta} \ll R_C$

So let $R_C = 10\left(\frac{R_F}{\beta}\right)$

$$V_{BE} + \frac{1}{\beta} I_C R_F + I_C\left(\frac{10}{\beta} R_F\right) = V_{CC}.$$

$$V_{BE} + I_C R_F\left(\frac{11}{\beta}\right) = V_{CC}.$$

To find V_{BE}: $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right).$

$$V_T \ln\left(\frac{I_C}{I_S}\right) + I_C R_F\left(\frac{11}{\beta}\right) = V_{CC}.$$

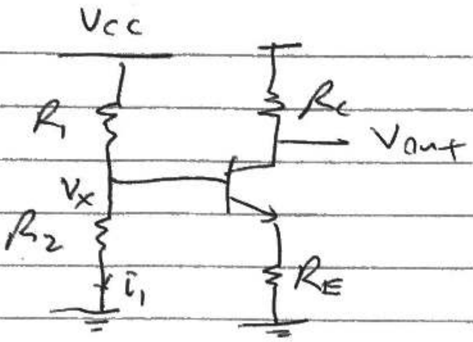
Now we can find R_C, R_F.

$$R_F = \frac{V_{CC} - V_T \ln\left(\frac{I_C}{I_S}\right)}{I_C \cdot \frac{11}{\beta}} = \frac{(2.5V) - (26mV)\ln\left(\frac{1mA}{5\times10^{-16}A}\right)}{(1mA)\cdot\frac{11}{100}}$$

$$= 16.0k\Omega.$$

$$R_C = \frac{10 R_F}{\beta} = \frac{R_F}{10} = 1.6k\Omega.$$

a, cont'd)



Want to make $I_B \ll I_1$ (current through R_1 and R_2)

so that we can approximate $V_x \approx V_{cc} \dfrac{R_2}{R_1 + R_2}$.

Thus, let $10 I_B = I_1 = \dfrac{V_{cc}}{R_1 + R_2}$.

Also know that $I_B = \dfrac{1}{\beta} I_c = \dfrac{1mA}{100}$.

$$\Rightarrow R_1 + R_2 = \frac{V_{cc}}{10 I_B} = \frac{V_{cc}}{\frac{10}{100} \cdot 1mA} = \frac{2.5\,V}{\frac{1mA}{10}}$$
$$= 25k\Omega.$$

Also know that $V_{BE} = V_T \ln\left(\dfrac{I_c}{I_S}\right)$, $V_x = I_E R_E + V_{BE}$
$$\approx I_c R_E + V_{BE}$$

and that $I_c R_E + I_c R_c + V_{BE} = V_{cc}$.
and that $\dfrac{V_{cc} - V_{out}}{R_c} = I_c = 1mA$.

Assume $V_{BE} = V_{CE}$ (on the edge of saturation), then

$V_x = V_{out}$, $\Rightarrow \dfrac{V_{cc} - V_x}{R_c} = I_c \Rightarrow \dfrac{V_{cc} - (I_E R_E + V_{BE})}{R_c} = I_c$.

then $V_{cc} = I_c R_c + I_c R_E + V_{BE}$.

Using the guideline: $R_E I_c > 4V_T$,

let $R_E = \dfrac{4V_T}{I_c} = \dfrac{4(26\,mV)}{1mA} = 104\,\Omega$

Solving for R_C:

$V_{CC} = I_C R_C + I_C R_E + V_{BE}$

$$\Rightarrow R_C = \frac{V_{CC} - I_C R_E - V_{BE}}{I_C}$$

$$= \frac{2.5V - (1mA)(104\Omega) - (26mV)\ln\left(\frac{1mA}{5\times10^{-16}A}\right)}{1mA}$$

$$= 1.66k\Omega$$

$$V_X \approx I_E R_E + V_{BE} = (1mA)(104\Omega) + (26mV)\ln\left(\frac{1mA}{5\times10^{-16}A}\right)$$

$$= 1.05V.$$

$$V_X = V_{CC} \cdot \frac{R_2}{R_1+R_2}$$

$$\Rightarrow \frac{R_2}{R_1+R_2} = \frac{V_X}{V_{CC}} = \frac{1.05V}{2.5V}$$

$$\Rightarrow R_2 = \left(\frac{1.05}{2.5}\right)(R_1+R_2) = \left(\frac{1.05}{2.5}\right)(25k\Omega)$$

$$= 10.5k\Omega$$

$$R_1 = 25k\Omega - 10.5k\Omega = 14.5k\Omega.$$

$$\boxed{\begin{aligned} R_1 &= 14.5k\Omega \\ R_2 &= 10.5k\Omega \\ R_C &= 1.66k\Omega \\ R_E &= 104\Omega \end{aligned}}$$

b) How much will V_{BE} change if V_{CC} is increased by 5%? Which circuit is less sensitive to changes?

a

Circuit (a) $V_{CC} \uparrow 5\% \approx V_{out} \uparrow 5\%$.

Since $\frac{1}{\beta} I_c$ current goes through the base,

$V_{BE} \uparrow \frac{1}{\beta} I_c \cdot 5\%$

sorry, ran out of time here.

Circuit (b): $V_{cc} \uparrow 5\% \Rightarrow \cancel{V_x \uparrow 5\%}$

$V_x \approx (0.41) V_{cc}$

so V_x increases by roughly $(0.05)(0.41)(2.5V) =$ 0.05125V

If we assume all of this is initially absorbed by V_{BE}, then:

$$I_c = I_s \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow I_c' = I_s \exp\left(\frac{V_{BE} + \Delta V_{BE}}{V_T}\right) = I_s \exp\left(\frac{V_{BE}}{V_T}\right) \times \exp\left(\frac{\Delta V_{BE}}{V_T}\right)$$

$$\exp\left(\frac{0.05125V}{26mV}\right) = 7.18.$$

So I_c increases by a large factor.

Thus $I_c R_E = (7.18)(1mA)(104\Omega) = 0.75V$.

Thus the voltage V_x at the base is decreased again

sorry, didn't finish these calculations.

c) Explain the mechanism by which each circuit inhibits changes in V_{BE} due to variation in V_{CC}.

10

Circuit (a) (self-biasing circuit) uses a negative feedback loop to regulate it's input. Increasing V_{BE} causes I_C to increase, which causes less current to go through the feedback loop to the base, which causes the voltage at the base to drop again..

Circuit (b) uses emitter degeneration to inhibit changes in V_{BE}. If V_{CC} increases, then the voltage at the base (call this V_X) also increases, but $V_X = V_{BE} + I_E R_E$. So the current I_E would also increase, "absorbing" much of the change in V_X so that V_{BE} doesn't change much.