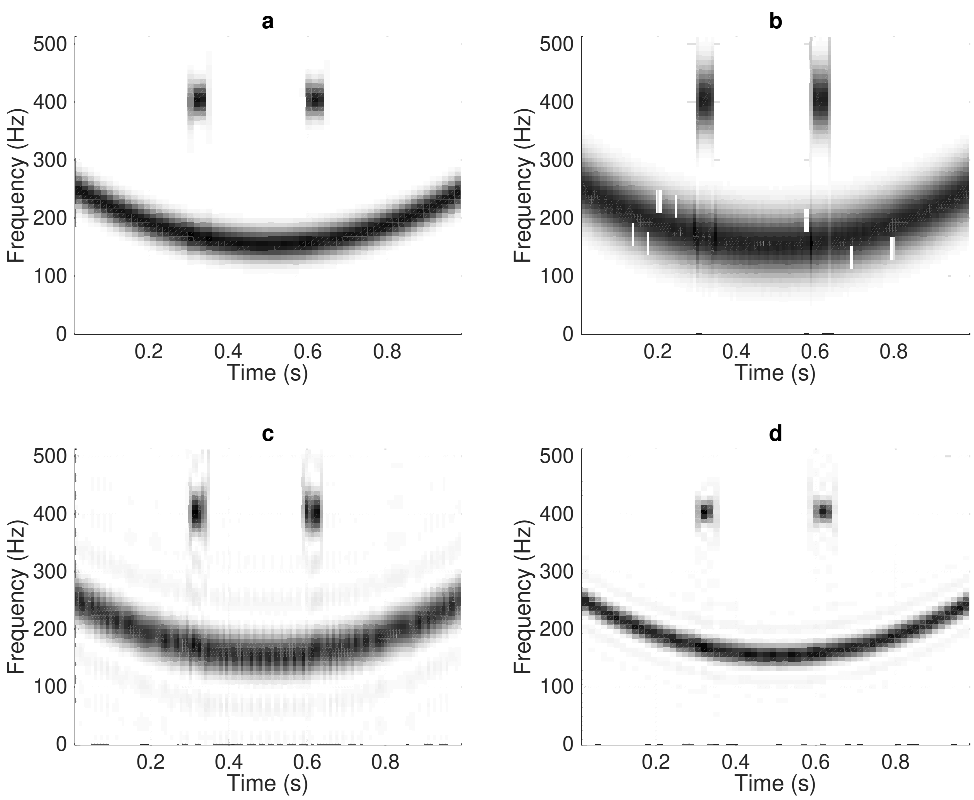
DSP Quiz 5
11/16/2020
Name:

Question 1

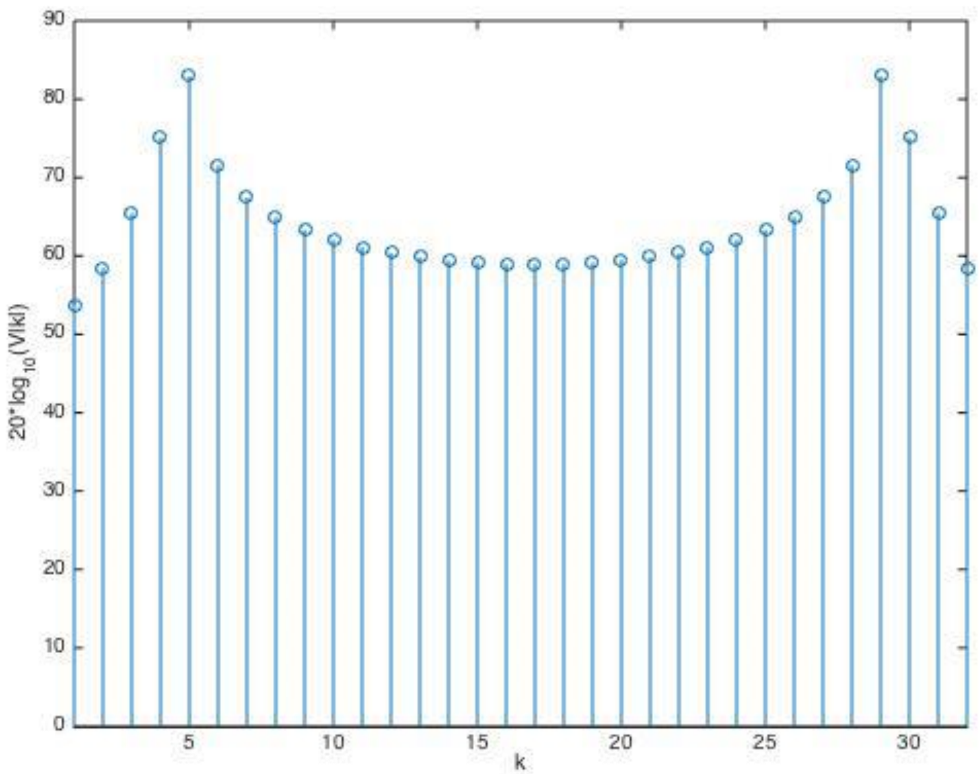
The following spectrograms were computed using either rectangular or Hamming windows, on the same signal. Answer the following questions, justify your answer completely.

a) Which spectrograms were computed with a rectangular window?
b) Which spectrograms have approximately the same frequency resolution?
c) What is the approximate time window of spectrogram a? Mark the plot if it helps indicate your answer
d) Write as detailed an equation as you can for the 'eye's, assuming that only pure sinusoidal tones were used to create them.



Question 2

Consider the following plot of the magnitude, in dB, of the DFT of a continuous time signal sampled at T= 10^{-3}. A 32 point DFT was taken using a rectangular window.



Listed below are 10 signals, one or more of which could have been the continuous time signal that produced the above plot. Indicate which signals could have been the input signal $x_c(t)$. Justify your answer completely

$x_1(t) = 1000 \cos(230*\pi*t)$	$x_6(t) = 1000 \exp(j*250*\pi*t)$
$x_2(t) = 1000 \cos(115*\pi*t)$	$x_7(t) = 10 \cos(250*\pi*t)$
$x_3(t) = 10 \exp(j*460*\pi*t)$	$x_8(t) = 1000(\cos(218.75*\pi*t)$
$x_4(t) = 1000 \exp(j*230*\pi*t)$	$x_9(t) = 10 \exp(j*200*\pi*t)$
$x_5(t) = 10 \exp(j*230*\pi*t)$	$x_{10}(t) = 1000 \exp(j(187.5*\pi*t)$

ECE310 – Quiz 5

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1. *Smiley face spectrograms*

   (a) Spectrograms (c) and (d) use the rectangular window (they have prominent sidelobe ripples). (a) and (b) use the Hamming window.

   (b) The spectrograms pairs ((a) and (d)) and ((b) and (c)) have similar frequency resolutions. Frequency resolution is the blurriness along the y-axis (frequency axis).

   (c) By a really rough estimate, blurring starts at $t = 0.29$s and ends at $t = 0.31$s, which would indicate a time window length of 0.02s. To be safe, I think the length of the time window lies somewhere in the range $[0.01, 0.05]$s.

   (d) By another rough estimate, it looks like the eyes start at 0.3s and 0.6s and last for 0.05s each.

$$y(t) = \begin{cases} \cos(2\pi(400)t), & |t - 0.325| < 0.025 \text{ or } |t - 0.625| < 0.025 \\ 0, & \text{else} \end{cases}$$

2. *DFT of a sinusoid*

   The peaks occur at $k = 4$ and $k = 28$ ($k = -4$ due to the wraparound). This means that the cosine lies somewhere between $k = 3$ and $k = 5$ on the DFT; looking at the way the peak is skewed it looks like it's slightly lower than $k = 4$. Use the formula:

$$\Omega_4 = 2\pi \frac{k}{NT} = 2\pi \frac{4}{(32)(0.001\text{s})} = 250\pi \text{ rad/s}$$

   We also note that $\Omega_3 = 187.5$rad/s, and halfway between Ω_3 and Ω_4 is 218.75rad/s. Using the intuition above, we know that the true frequency lies in the non-inclusive range $(218.75, 250)$. ($\Omega = 218.75$ wouldn't be valid, because then the two frequencies at $k = 3$ and $k = 4$ would be equal height due to the symmetry of the DFT of the cosine, and similarly $\Omega = 250$ wouldn't be valid either). That eliminates all but x_1, x_4, and x_5. Due to the asymmetry of the DFT of a complex signal (i.e., look at Euler's formula, the sin has a phase shift that makes it asymmetric), we can also eliminate x_4 and x_5. Thus, x_1 is the only signal that could have this DFT.