DSP Quiz #4
Discrete Fourier Transform
11/23/2020

Consider the following finite length sequence:

$$x[n] = 2\delta[n] + \delta[n-1] + \delta[n-3]$$

a) Compute the 5-point DFT $X[k]$
b) Compute the 5-point DFT $Y[k] = X[k]^2$
c) Compute the inverse 5-point DFT of $Y[k]$ to find the sequence $y[n]$ for n = 0, 1, 2, 3, 4
d) If N-point DFTs are used in the two-step procedure, how should we choose N so that $y[n] = x[n] * x[n]$ for $0 \le n \le N-1$?
e) Repeat steps a-c with the value you found in part D.

DSP Quiz 4

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$x[n] = 2\delta[n] + \delta[n-1] + \delta[n-3]$, $N = 5$

a) DFT is linear, and DFT of $\delta[n-d]$ is W_N^{dk}.

$$\Rightarrow X[k] = \underbrace{2W_5^0}_{=2} + W_5^k + W_5^{3k}$$

b) Compute 5 pt DFT $Y[k] = (X[k])^2$

$$Y[k] = (X[k])^2 = \left(2 + W_5^k + W_5^{3k}\right)\left(2 + W_5^k + W_5^{3k}\right)$$

$$= 4 + 2W_5^k + 2W_5^{3k} + 2W_5^k + W_5^{2k} + W_5^{4k} + 2W_5^{3k} + W_5^{4k} + \underbrace{W_5^{6k}}_{=W_5^k}$$

← wrap around due to aliasing

$$= 4 + 5W_5^k + W_5^{2k} + 4W_5^{3k} + 2W_5^{4k}$$

c) $g[n] = 4\delta[n] + 5\delta[n-1] + \delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$

d) To avoid aliasing, choose N s.t. N is the length of the convolution, i.e., $N =$ [redacted]. $4+4-1 = 7$

(Note: I did $4+4-1$ rather than $5+5-1$ because the support of the original signal is length 4, not 5).

e) $N = 7$.

$$X[k] = \underbrace{2W_7^0}_{=2.} + W_7^k + W_7^{3k}.$$

$$Y[k] = 4 + 4W_7^k + W_7^{2k} + 4W_7^{3k} + 2W_7^{4k} + W_7^{6k}$$

(almost the same as previous, but the W_7^{6k} doesn't "wrap around")

$$y[n] = 4\delta[n] + 4\delta[n-1] + \delta[n-2] + 4\delta[n-3] + 2\delta[n-4] + \delta[n-6]$$

$$= x[n] * x[n]$$

(I checked the convolution of x with itself on MATLAB)