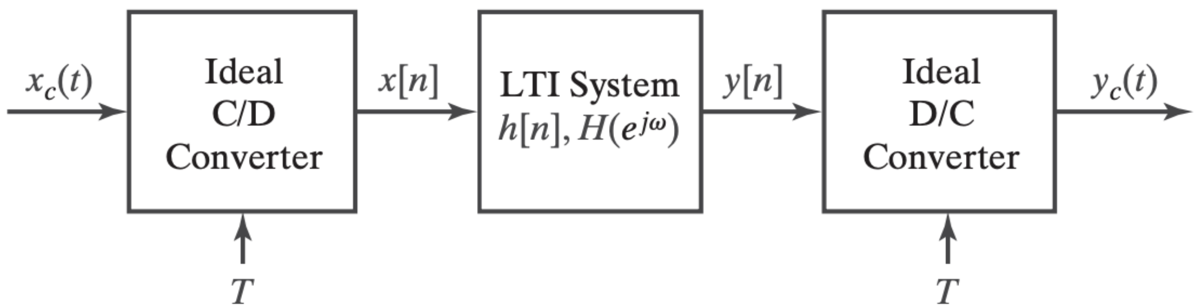
DSP Fall 2020
Quiz #3 makeup, Filter Design
Name:

Question 1:

Consider the system below, where $H(e^{j\omega})$ is a bandpass filter, with passband ripple $\delta_1 = .001$ and stop band ripple $\delta_2 = .001$ and band edges $\omega_{s1} = .2\pi$ $\omega_{p1} = .4\pi$, $\omega_{p2} = .6\pi$. $\omega_{p2} = .8\pi$. The sampling rate for the idea C/D and D/C is $1/T = 10,000$ samples per second. Furthermore, it is known that the filter $H(e^{j\omega})$ has a maximum group delay of 34 samples.



a) (1 point) What property should the input signal have so that the overall system behaves as an LTI system with $Y_c(j\Omega) = H_{eff}(j\Omega)\, X_c(j\Omega)$?
b) (1 point) For the conditions found in part a), sketch $|H_{eff}(j\Omega)|$, include as much detail as possible.
c) (2 points) Based on the information given, could the filter $H(e^{j\omega})$ have been designed using the impulse invariance method? If so, specify the analog prototype filter $H_{ap}(j\Omega)$. If not, explain why not.
d) (2 points) Based on the information given, could the filter $H(e^{j\omega})$ have been designed using the blinear transformation method? If so, specify the analog prototype filter $H_{ap}(j\Omega)$. If not, explain why not.
e) (2 points) Based on the information given, could the filter $H(e^{j\omega})$ have been designed using the Kaiser window method? If so, specify the ideal impulse response used in the design. If not, explain why not.
f) (2 points) Based on the information given, could the filter $H(e^{j\omega})$ have been designed using the Park-McLellan algorithm? If so, specify possible values for the group delay. If not, explain why not.

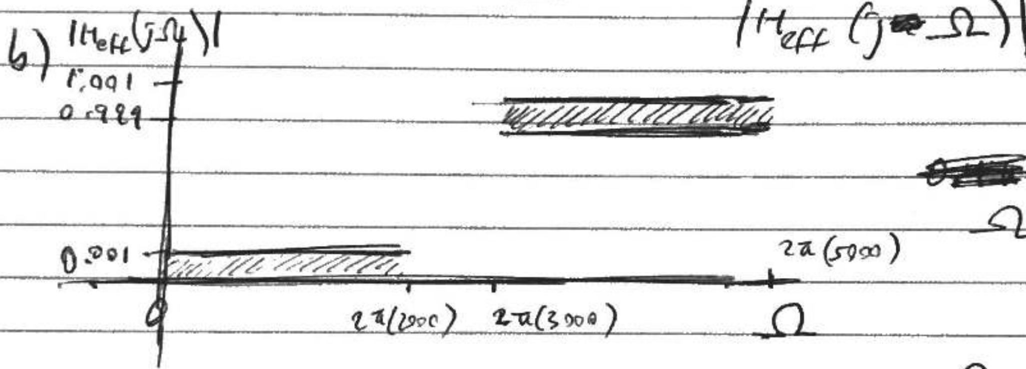
$H(e^{j\omega})$ is a highpass filter, $\delta_p = 0.001$, $\delta_s = 0.001$, $\omega_s = 0.4\pi$, $\omega_p = 0.6\pi$.

$T_s = \frac{1}{10000}\, s \Rightarrow \Omega_s = 2\pi(10000)\, \frac{rad}{s}$

$H(e^{j\omega})$ has maximum group delay of 14 samples

a) If there is any aliasing, then the system will not be LTI. Thus the bandwidth of $X_c(t)$ should be $\leq$ Nyquist bandwidth, i.e., should be less than $2\pi(5000)\, \frac{rad}{s}$.

b)



$\omega = \Omega T$

$\Omega_s = \frac{\omega_s}{T} = \frac{0.4\pi}{\frac{1}{10000}}$

$= 2\pi(2000)$

$\Omega_p = \frac{\omega_p}{T} = \frac{0.6\pi}{\frac{1}{10000}} = 2\pi(3000)$

c) Impulse invariance cannot be used for a HPF due to aliasing (i.e., higher frequencies will alias into lower frequencies). However, bilinear transform would work b/c there's no aliasing there.

d) bilinear transformation can be used because it doesn't introduce aliasing.

analog prototype filter:

$$\Omega = \frac{2}{T_d}\tan\left(\frac{\omega}{2}\right)$$

$$\begin{cases} 0.999 \le |H(e^{j\omega})| \le 1.001 & \text{when } 0.6\pi \le \omega \le \pi. \\ |H(e^{j\omega})| \le 0.001 & \text{when } 0 \le \omega \le 0.4\pi \end{cases}$$

$$\Downarrow$$

$$\begin{cases} 0.999 \le |H_{ap}(j\Omega)| \le 1.001 & \text{when } 2(10000)\tan\left(\frac{0.6\pi}{2}\right) \le \Omega \le 2(10000)\tan\left(\frac{\pi}{2}\right) \\ |H_{ap}(j\Omega)| \le 0.001 & \text{when } 0 \le \Omega \le 2(10000)\tan\left(\frac{0.4\pi}{2}\right) \end{cases}$$

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e) No ~~Yes,~~ Kaiser windows can generally represent HPFs by subtracting one LPF window from another

$\boxed{\Delta\omega = 0.2\pi}$

$$A = -20\log_{10}\delta = -20\log_{10}(0.001) = 60$$

$$\beta = 0.1102(60 - 8.7)$$

$$M = \frac{A-8}{2.285\Delta\omega} = \frac{60-8}{(2.285)(0.2\pi)} = 36.2 \rightarrow 37$$

$$h[n] = \frac{\sin\left(\pi\left(n - \frac{36}{2}\right)\right)}{\pi\left(n - \frac{36}{2}\right)} - \frac{\sin\left(0.5\pi\left(n - \frac{36}{2}\right)\right)}{\pi\left(n - \frac{36}{2}\right)}$$

The group delay of this Kaiser window is $\frac{M}{2} = 18 > 14$

So this cannot be the filter.

~~Yes~~ No, we can compute this like for the Kaiser window.

f). Using equation 7.117:

$$M = \frac{-10\log_{10}(\delta_1\delta_2) - 13}{2.324\,\Delta\omega}$$

$$= \frac{-10\log_{10}((0.001)(0.001)) - 13}{2.324(0.2\pi)}$$

$$\approx 32.2 \xrightarrow[\text{up}]{\text{round}} 33$$

group delay $= \frac{33}{2} \approx 17$.

but group delay is too high again $(17 > 14)$.