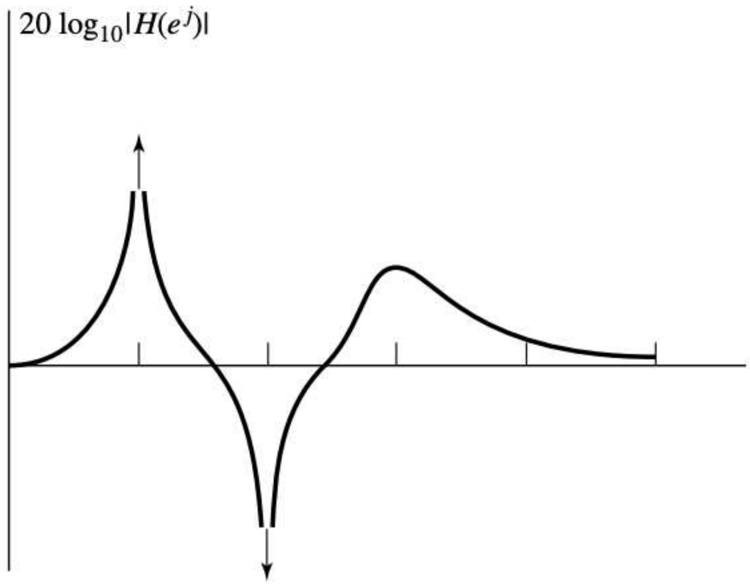
DSP Quiz #2
10/7/20
Name:

Question 1: Consider a causal linear time-invariant system with system function H(z) and real impulse response. H(z) is evaluated for $z = e^{j\omega}$ and is shown in the figure below:



1 point each:

a) Carefully sketch a pole–zero plot for H(z) showing all information about the pole and zero locations that can be inferred from the figure.
b) What can be said about the length of the impulse response? Justify your answer
c) Is this a linear phase system? How can you tell?
d) Is this system stable? How can you tell?
e) Is this an all-pass system? How can you tell?
f) Is this a minimum-phase system? How can you tell?

Question 2:

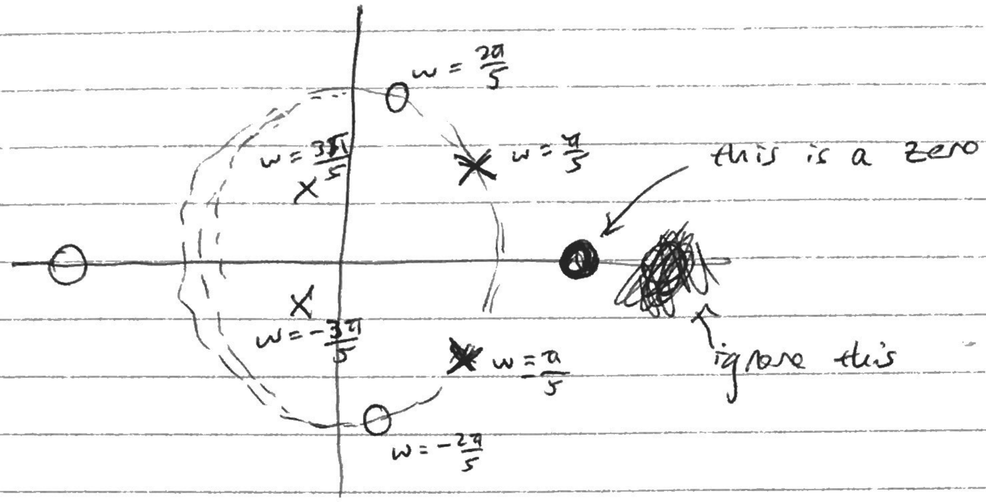
Consider the following system function:

$$H(z) = \frac{(1 - .5z^{-1})(1 + 4z^{-2})}{1 - .64z^{-2}}$$

a) (1 point) Sketch the pole-zero plot for H(z)
b) (1 point)Write the difference equation that relates the input and the output of this system
c) (2 points)Find expressions for a minimum-phase system such that $H_1(z)$ and an all-pass system $H_{ap}(z)$ such that $H(z) = H_1(z)H_{ap}(z)$

ECE 310 Quiz 2 Jonathan Lam 10/8/20

1) a)



We can tell from this magnitude response plot is that there is a pole @ $w = \pm\frac{\pi}{5}$, a zero @ $w = \pm\frac{2\pi}{5}$. I'm assuming from the graph that these points on the magnitude response graphs are asymptotes, i.e., the pole and the zero lie exactly on the unit circle

Since the magnitude response at $w = 0$ is ≈ 0 dB, this means there must be some other zero on the plot to balance out the poles near 0. There might also be some poles $\approx w = \pm\frac{3\pi}{5}$ to cause the magnitude response to rise again after reaching the zero, and again there might be a zero near $w = \pi$ to bring the magnitude response down to near 0 dB again (i.e., the right end of the magnitude response plot).

b) IIR: has poles that are not at the origin.

c) not linear phase because it is IIR.

d) not stable, poles on the unit circle.

e) not all-pass, magnitude response is not constant

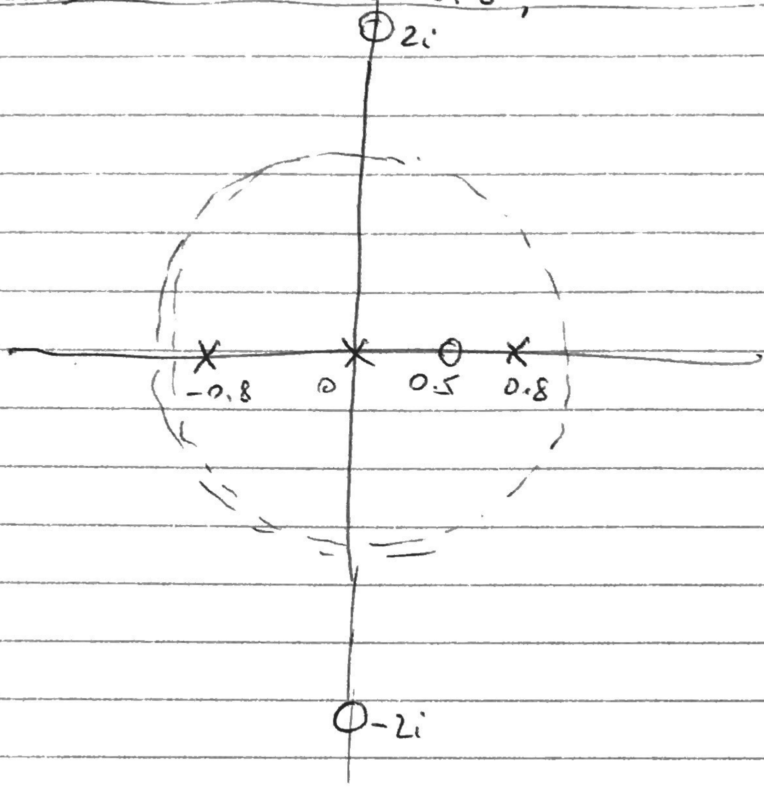
f) not minimum phase, not all poles and zeroes are in the unit circle.

2.) $H(z) = \dfrac{(1-0.5z^{-1})(1+4z^{-2})}{1-0.64z^{-2}}$

zeros @ $z = 0.5, \pm 2i$

poles @ $z = \pm 0.8$, ~~[illegible]~~ 0

a)



b) $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{1+4z^{-2}-0.5z^{-1}-2z^{-3}}{1-0.64z^{-2}}$

$$Y(z)(1-0.64z^{-2}) = X(z)(1+4z^{-2}-0.5z^{-1}-2z^{-3})$$

$\Downarrow$ ~~inverse z-transform~~

$$y[n] - 0.64y[n-2] = x[n] - 0.5x[n-1] + 4x[n-2] - 2x[n-3]$$

c) only the $\pm 2i$ zeros are outside the unit circle, have to "flip" them inside the circle to $\pm\frac{1}{2}i$. (conjugate ~~&~~ reciprocal).

$$(1+4z^{-2}) = (1+2iz^{-1})(1-2iz^{-1})$$

$$= \left(\left(\frac{1+2iz^{-1}}{1-2iz}\right)(1-2iz)\right)\left(\left(\frac{1-2iz^{-1}}{1+2iz}\right)(1+2iz)\right)$$

$$= \left(\frac{1+2iz^{-1}}{1-2iz}\cdot\frac{1-2iz^{-1}}{1+2iz}\right)(1-2iz)(1+2iz)$$

$$= \left(\frac{1+4z^{-2}}{1+4z^{2}}\right)(1+4z^{2}) = \underbrace{\left(\frac{1+4z^{-2}}{1+\frac{1}{4}z^{-2}}\right)}_{\text{all-pass}}\underbrace{\left(1+\frac{1}{4}z^{-2}\right)}_{\text{minimum phase}}$$

plugging back into the original equation:

10/8/20

$$H(z) = \underbrace{\left(\frac{1+4z^{-2}}{1+\frac{1}{4}z^{-2}}\right)}_{\text{all-pass}} \underbrace{\left(\frac{(1-0.5z^{-1})(1+\frac{1}{4}z^{-2})}{1-0.64z^{-2}}\right)}_{\text{minimum phase}}$$