ECE310 – Pset 3

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Problems: 5.11, 5.12, 5.28 (use MATLAB for part c), 5.34, 5.45

5.11 *The system function of an LTI system has the pole–zero plot shown in Figure P5.11 Specify whether each of the following statements is true, is false, or cannot be determined from the information given.*

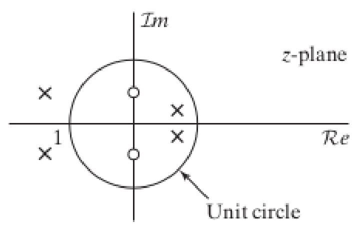

Figure 5.11

a) *The system is stable.*

   Cannot be determined from the information given. (3 possible ROCs, only one including unit circle is stable.)

b) *The system is causal.*

   Cannot be determined from the information given. (3 possible ROCs, only outermost one is causal.)

c) *If the system is causal, then it must be stable.*

   False. See reasoning above.

d) *If the system is stable, then it must have a two-sided impulse response.*

   True. The only stable ROC is bounded on both inside and outside by poles, which means that it has a two-sided impulse response.

5.12 *A discrete-time causal LTI system has the system function*

$$H(z) = \frac{\left(1 + 0.2z^{-1}\right)\left(1 - 9z^{-2}\right)}{1 + 0.81z^{-2}}$$

a) *Is the system stable?*

Since this system is causal, and the only poles are at $\pm 0.9j$, the ROC is $|z| > 0.9$ includes the unit circle and is thus stable.

b) *Determine expressions for a minimum-phase system $H_1(z)$ and an all-pass system $H_{ap}(z)$ such that*

$$H(z) = H_1(z) H_{ap}(z)$$

The only poles or zeros outside of the unit circle are the zeros at $z = \pm 3$, so we need to "flip" these into the unit circle to generate the minimum-phase component. I.e., we have to decompose the $\left(1 - 9z^{-2}\right)$ term:

$$\begin{aligned}
1 - 9z^{-2} &= (1 + 3z^{-1})(1 - 3z^{-1}) \\
&= \left[\left(\frac{1 + 3z^{-1}}{3 + z^{-1}}\right)\left(3\left(1 + \frac{1}{3}z^{-1}\right)\right)\right]\left[\left(\frac{1 - 3z^{-1}}{-3 + z^{-1}}\right)\left(-3\left(1 - \frac{1}{3}z^{-1}\right)\right)\right] \\
&= \left[\frac{1 + 3z^{-1}}{3 + z^{-1}}\frac{1 - 3z^{-1}}{-3 + z^{-1}}\right]\left[-9\left(1 + \frac{1}{3}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)\right] \\
&= \frac{1 - 9z^{-2}}{-9 + z^{-2}}\left(-9\left(1 - \frac{1}{9}z^{-2}\right)\right) \\
H(z) &= \frac{1 + 0.2z^{-1}}{1 + 0.81z^{-2}}\left(-9\left(1 - \frac{1}{9}z^{-2}\right)\right)\left(\frac{1 - 9z^{-2}}{-9 + z^{-2}}\right) \\
&= \left[-9\frac{\left(1 + 0.2z^{-1}\right)\left(1 - \frac{1}{9}z^{-2}\right)}{1 + 0.81z^{-2}}\right]\left[\frac{1 - 9z^{-2}}{-9 + z^{-2}}\right] \\
&= H_1(z) H_{\text{ap}}(z)
\end{aligned}$$

5.28 *A causal LTI system has the system function*

$$H(z) = \frac{\left(1 - e^{j\pi/3}z^{-1}\right)\left(1 - e^{-j\pi/3}z^{-1}\right)\left(1 + 1.1765z^{-1}\right)}{\left(1 - 0.9e^{j\pi/3}z^{-1}\right)\left(1 - 0.9e^{-j\pi/3}z^{-1}\right)\left(1 + 0.85z^{-1}\right)}$$

a) *Write the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of this system.*

$$\left(1 - e^{j\pi/3}z^{-1}\right)\left(1 - e^{-j\pi/3}z^{-1}\right)\left(1 + 1.1765z^{-1}\right) = 1 + 0.1765z^{-1} - 0.1765z^{-2} + 1.1765z^{-3}$$

$$\left(1 - 0.9e^{j\pi/3}z^{-1}\right)\left(1 - 0.9e^{-j\pi/3}z^{-1}\right)\left(1 + 0.85z^{-1}\right) = 1 - 0.05z^{-1} + 0.045z^{-2} + 0.6885z^{-3}$$

$$\begin{aligned}
&y[n] - 0.05y[n-1] + 0.045y[n-2] + 0.6885y[n-3] \\
&= x[n] + 0.1765x[n-1] - 0.1765x[n-2] + 1.1765x[n-3]
\end{aligned}$$

b) *Plot the pole–zero diagram and indicate the ROC for the system function.*

See Figure 1; Since the system is causal, the ROC is the outermost region, i.e., $|z| > 0.9$.

c) *Make a carefully labeled sketch of $|H(e^{j\omega}|$. Use the pole–zero locations to explain why the frequency response looks as it does.*

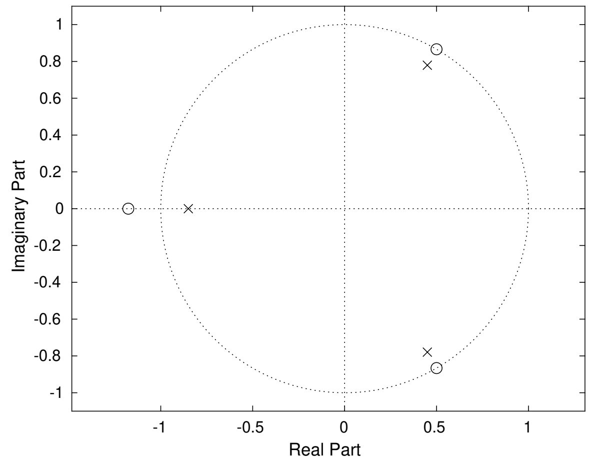

Figure 1: Pole-zero plot of $H(z)$.

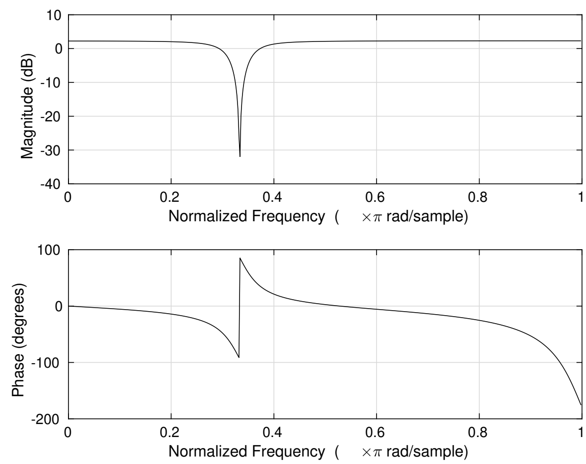

Figure 2: Frequency response of $H(z)$.

See Figure 2. (Using MATLAB's freqz generates both magnitude and phase response, and only for positive normalized frequency (it should be symmetric for negative frequencies); the top chart is the answer to this question.)

d) *State whether the following are true or false about the system:*

i) *The system is stable.*
True. The ROC includes the unit circle (all poles lie inside the unit circle).

ii) *The impulse response approaches a nonzero constant for large n.*
False. Since the system is stable, the impulse response must approach zero when n is large.

iii) *Because the system function has a pole at angle $\pi/3$, the magnitude of the frequency response has a peak at approximately $\omega = \pi/3$.*

False. The system also has a zero at $\omega = \pi/3$ (and actually on the unit circle, whereas the pole is slightly off the unit circle), so the magnitude of the frequency response is actually zero at $\omega = \pi/3$.

iv) *The system is a minimum-phase system.*
False. There is a zero at $z = -1.1765$ (outside of the unit circle).

v) *The system has a causal and stable inverse.*
False. Since it is not minimum phase, its inverse will be either non-causal or unstable.

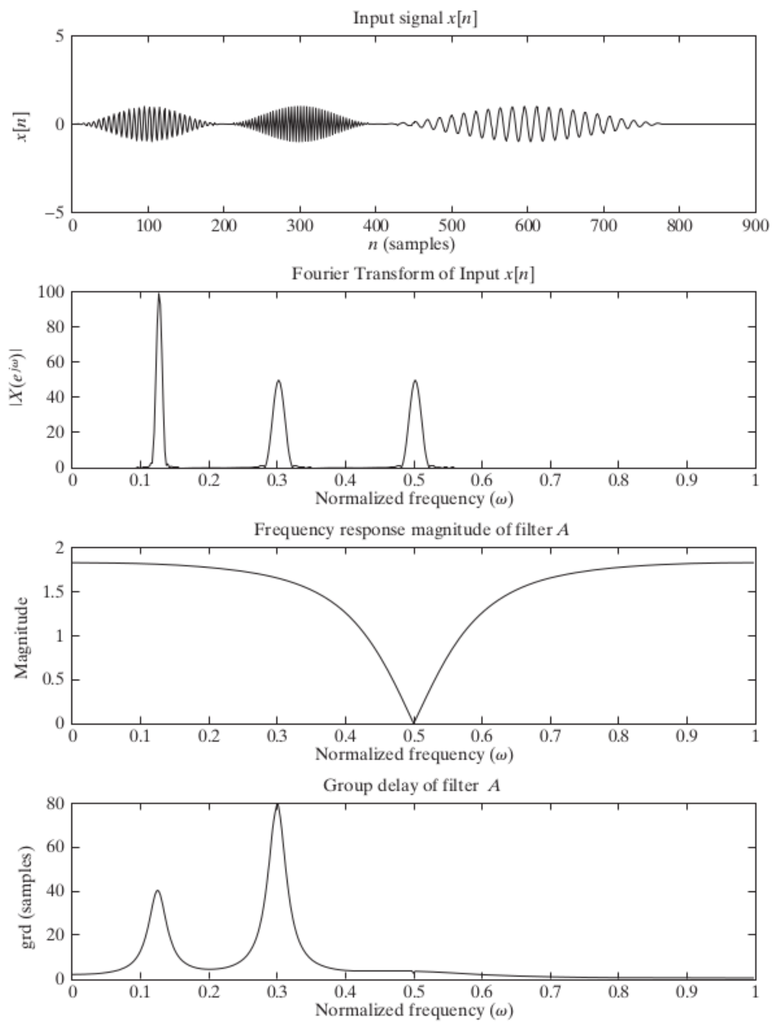

Figure 5.34-1: The input signal and the filter frequency response.

5.34 *A discrete-time LTI system with input $x[n]$ and output $y[n]$ has the frequency response magnitude and group delay functions shown in Figure P5.34-1. The signal $x[n]$, also shown in*

Figure P5.34-1, is the sum of three narrowband pulses. In particular, Figure P5.34-1 contains the following plots:

- $x[n]$
- $|X(e^{j\omega})|$, *the Fourier transform magnitude of a particular input* $x[n]$
- *Frequency response magnitude plot for the system*
- *Group delay plot for the system*

In Figure P5.34-2 you are given four possible output signals, y i [n] i = 1, 2, . . . , 4. Determine which one of the possible output signals is the output of the system when the input is x[n]. Provide a justification for your choice.

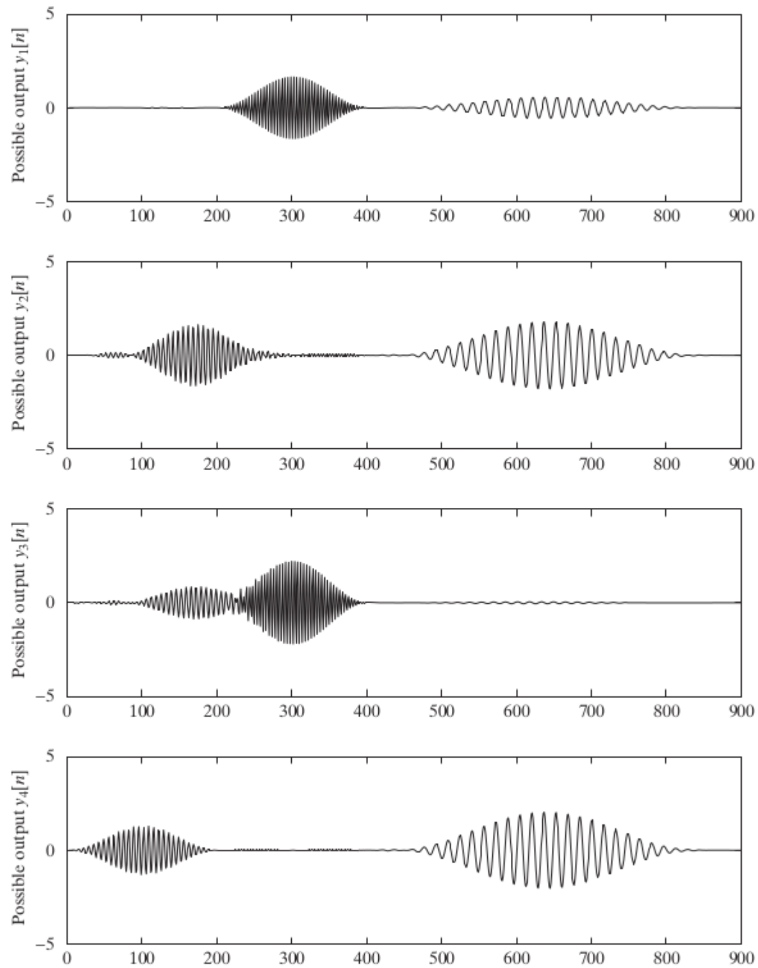

Figure 5.34-2: Possible output signals.

The input signal consists of three pulses with different frequencies. The first pulse is at a medium frequency (0.3 on a normalized scale), the second is at a higher frequency (0.5), and the last is at the lowest frequency (≈ 0.12).

The filter is a bandstop filter with the stopband frequency at 0.5, so the high frequency pulse should not be seen in the output. Both of the other pulses should not be significantly attenuated; the 0.3 frequency pulse will be scaled by less than the 0.1 frequency pulse.

The filter also has a high group delay (80 samples) for the 0.3 frequency pulse, and a lower group delay for the 0.1 frequency pulse (40 samples), which means that that the distance between the two pulses will be smaller in the output.

The one that matches this best is $y_2[n]$, which has the first pulse shifted right by approximately 80 samples, the second pulse obliterated by the band-stop filter, and the last pulse shifted right by approximately 40 samples and slightly scaled up more than the first pulse.

5.45 *The pole–zero plots in Figure P5.45 describe six different causal LTI systems.*

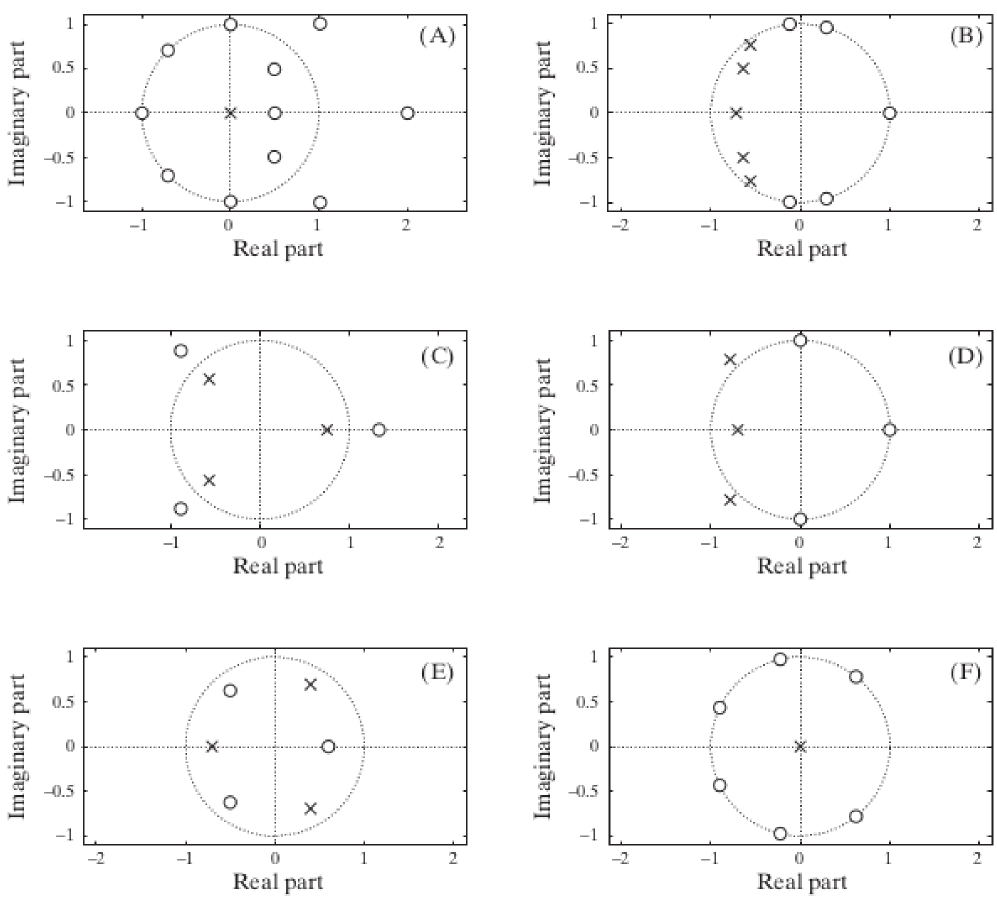

Figure 5.45

Answer the following questions about the systems having the above pole–zero plots. In each case, an acceptable answer could be none or all.

a) *Which systems are IIR systems?*

(B), (C), (D), (E). IIR systems have poles at places other than the origin and infinity.

b) *Which systems are FIR systems?*

(A), (F). An FIR system only has poles at the origin or infinity.

c) *Which systems are stable systems?*

(A), (B), (C), (E), (F). A causal system is stable if all its poles lie inside the unit circle.

d) *Which systems are minimum-phase systems?*

(E). A causal system is minimum phase if its poles and zeros all lie inside the unit circle.

e) *Which systems are generalized linear-phase systems?*

(A), (F). Minimum phase systems have to be FIR and have to be symmetric, and have zeros only on the unit circle or in reciprocal pairs.

f) *Which systems have* $|H(e^{j\omega})| =$*constant for all* ω*?*

(C). A system is all-pass if it has poles and zeros at conjugate reciprocal pairs. (We can't really tell if the points in C are truly at conjugate reciprocal distances away from the unit circle but it appears to be so.)

g) *Which systems have corresponding stable and causal inverse systems?*

(E). This is equivalent to being minimum phase.

h) *Which system has the shortest (least number of nonzero samples) impulse response?*

(F). System F only has seven zeros or poles, corresponding to seven nonzero samples in the impulse response. (System A has more, and the rest are IIR).

i) *Which systems have lowpass frequency responses?*

(A), (F). A lowpass frequency response will occur when a system has a high magnitude response near zero frequency (i.e., no zeros too close to $z = 1$) and low magnitude response everywhere else (i.e., zeros along the rest of the circle are fine).

j) *Which systems have minimum group delay?*

(E). Minimum-phase systems minimize phase distortion (phase response is roughly constant), and thus also minimize group delay.