ECE241 – Quiz 3

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May 15, 2020

BJT Voltage Amplifier

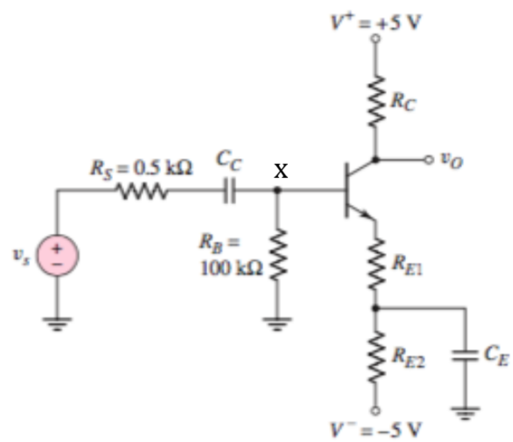

Figure 1: Common emitter stage

Assume that C_1, C_2 are large enough so that they act as open to DC and short to AC.

Common emitter stage voltage gain

Show that the voltage gain of the common emitter stage can be approximated as:

$$|A_v| = \frac{R_C}{\frac{1}{g_m} + R_{E_1}}$$

Consider only the common-emitter transistor implementation (and ignoring the input signal). Use the small signal model for the BJT, treating capacitor are

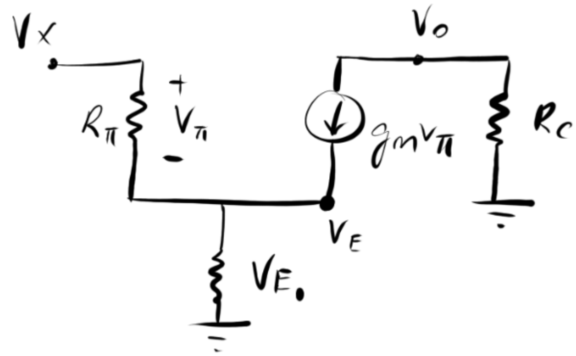

Figure 2: Small signal model of common emitter stage

short circuits. Perform KCL on the the two nodes.

$$\frac{V_x - V_E}{R_\pi} + \frac{0 - V_E}{R_{E_1}} + g_m V_\pi = 0$$
$$g_m V_\pi + \frac{V_o}{R_C} = 0$$

Since the base current is small relative to the amplified current, approximate that the first term of the first equation, $V_\pi / R_\pi \approx 0$. Then:

$$\begin{aligned}
V_x\left(g_m\right) &= V_E \frac{1}{R_{E_1}} + g_m \\
V_o &= g_m R_C V_\pi = g_m R_C \left(V_x - V_E\right) \\
&= g_m R_C V_x \left(1 - \frac{g_m}{\frac{1}{R_{E_1}} + g_m}\right) \\
&= g_m R_C V_x \left(\frac{\frac{1}{R_{E_1}} + g_m - g_m}{\frac{1}{R_{E_1}} + g_m}\right) \\
&= \frac{R_C V_x}{\frac{1}{g_m} + R_{E_1}} \\
\Rightarrow \left|A_v\right| = \frac{V_o}{V_x} &= \frac{R_C}{\frac{1}{g_m} + R_{E_1}}
\end{aligned}$$

which is the desired result. The attenuation factor is related to this voltage gain (the higher the gain, the lower the attenuation.) It can be seen that it is dependent on the current flowing the transistor, as $g_m \propto I_C$: a higher current causes a larger gain.

Circuit design to match specifications

Design the circuit such that $V_{CEQ} = 4\text{V}$ *and to amplify a* 12mV *sinusoidal signal from a microphone, having an output resistance of* $0.5\text{k}\Omega$*, to a* 0.4V *sinusoidal output signal. Approximate* $|A_v| \approx R_C/R_E$*. Assume that the transistor used in the design has nominal values of* $\beta = 100$*,* $V_{BE(ON)} = 0.7\text{V}$*, and* $I_s = 4 \times 10^{-16}\text{V}$.

Start with the large signal model to find R_C and R_E.

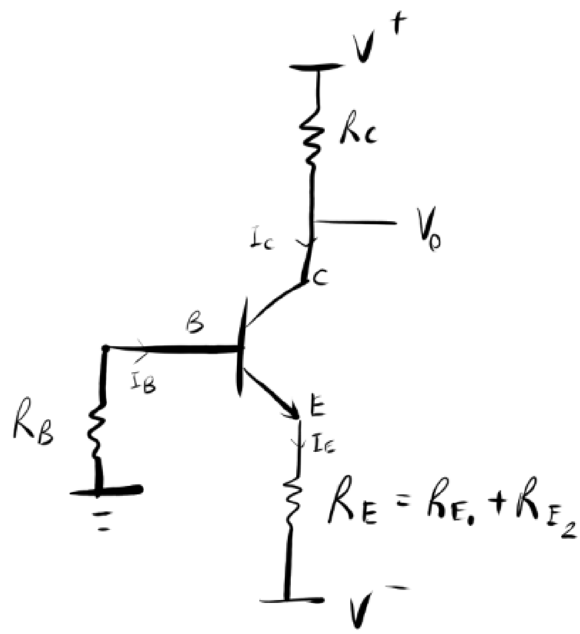

Figure 3: Large signal model

$$I_C = I_s \exp\left(\frac{V_{BE}}{V_T}\right) = 1.97 \times 10^{-4}\text{A}$$
$$I_B = \frac{I_C}{\beta} = 1.97 \times 10^{-6}\text{A}$$
$$I_E \approx I_C$$
$$V_B = -R_B I_B = 0.197\text{V}$$
$$V_E = V_B - V_{BE} = -0.897\text{V}$$
$$V_C = V_E + V_{CEQ} = 3.103\text{V}$$
$$R_C = \frac{V^+ - V_C}{I_C} = \frac{5\text{V} - 3.303\text{V}}{1.97 \times 10^{-4}\text{A}} = 9629\Omega$$
$$R_E = \frac{V_E - V^-}{I_E} = 20827\Omega = R_{E_1} + R_{E_2}$$

Now we need to find R_{E_1} and R_{E_2} (such that their sum equals R_E). We turn to the small signal model, where $g_m = \frac{I_C}{V_T}$ and $R_\pi = \frac{\beta}{g_m}$. By KCL:

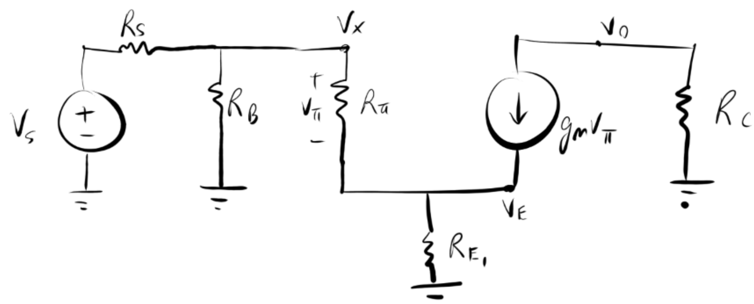

Figure 4: Small signal model

$$\frac{V_s - V_x}{R_s} + \frac{0 - V_x}{R_B} + \frac{V_E - V_x}{R_\pi} = 0$$
$$\frac{V_x - V_E}{R_\pi} + \frac{0 - V_E}{R_{E_1}} + g_m(V_x - V_E) = 0$$

Rearrange these equations to get V_s in terms of V_x.

$$\begin{aligned}
V_s\left(\frac{1}{R_s}\right) &= V_x\left(\frac{1}{R_s}+\frac{1}{R_B}+\frac{1}{R_\pi}\right) - V_E\left(\frac{1}{R_\pi}\right)\\
V_E\left(\frac{1}{R_\pi}+\frac{1}{R_{E_1}}+g_m\right) &= V_x\left(\frac{1}{R_\pi}+g_m\right)\\
\frac{V_s}{V_x} &= R_s\left(\frac{1}{R_s}+\frac{1}{R_B}+\frac{1}{R_\pi}-\frac{1}{R_\pi}\left(\frac{\frac{1}{R_\pi}+G_m}{\frac{1}{R_\pi}+\frac{1}{R_{E_1}+g_m}}\right)\right)\\
&= 1+\frac{R_s}{R_\pi}+\frac{R_s}{R_B}-\frac{R_s}{R_\pi}\left(\frac{1+R_\pi g_m}{1+\frac{R_\pi}{R_{E_1}}+g_m R_\pi}\right)
\end{aligned}$$

Since we know that $|A_V| \approx \frac{R_C}{R_{E_1}}$ from the previous section, and since we know that the desired output $V_o = 0.4\text{V}$ when $V_s = 0.012\text{V}$, then we can solve for R_{E_1}:

$$A_V = \frac{R_C}{R_{E_1}} = \frac{V_o}{V_x} = \frac{V_s}{V_x}\frac{V_o}{V_s} = \left[1+\frac{R_s}{R_\pi}+\frac{R_s}{R_B}-\frac{R_s}{R_\pi}\left(\frac{1+R_\pi g_m}{1+\frac{R_\pi}{R_{E_1}}+g_m R_\pi}\right)\right]\frac{0.4\text{V}}{0.012\text{V}}$$

The only unknown here is R_{E_1}. Plugging into a graphing calculator (desmos.com) and visually solving the expression, we get $R_{E_1} = 286\Omega$. In summary:

$$\begin{aligned}
R_C &= 9629\Omega\\
R_{E_1} &= 286\Omega\\
R_{E_2} &= 20827\Omega - 286\Omega = 20541\Omega
\end{aligned}$$

Diode-connected transistor

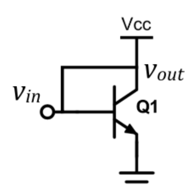

Figure 5: Diode-connected transistor

You may assume that the transistor, with common-emitter current gain β, is in forward active mode and has the following small signal parameters: r_π, g_m.

Small signal input impedance

Determine the small signal input impedance.

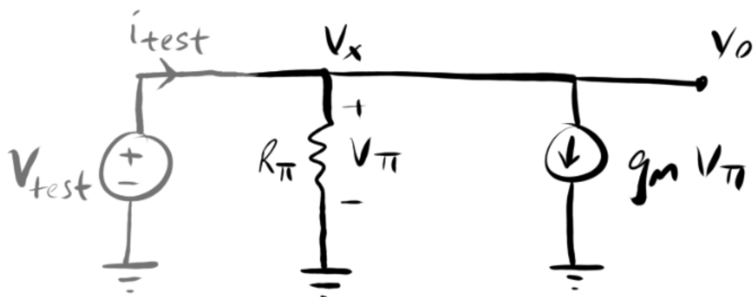

Figure 6: Small-signal model with test voltage

Use KCL at the only non-ground node (which is at voltage $V_\pi = V_o = V_{test}$):

$$I_{test} = \frac{V_\pi}{R_\pi} + g_m V_\pi = \frac{V_\pi}{\beta/g_m} + g_m V_\pi = g_m V_\pi \left(\frac{\beta+1}{\beta}\right) = V_{test} g_m \left(\frac{\beta+1}{\beta}\right)$$

$$R_{in} = \frac{V_{test}}{I_{test}} = \frac{\beta}{g_m(\beta+1)} \approx \frac{1}{g_m}$$

Since $g_m = I_C/V_T$, the input impedance is roughly inversely proportional to the current $I \approx I_C$ through the BJT, which means that it will maintain a roughly constant voltage across it ($V = IR_{in}$, and I and R_{in} are roughly proportional), similar to a diode.

Justify the name

In this circuit configuration, Q_1 is referred to as a diode-connected transistor. Justify!

Just by looking at the circuit, we can see that the collector and base are connected, so the voltage $V_{CB} = 0$ and there is no current flowing between those terminals. Thus, the "input" terminal is the base and the output is the emitter, which is just a PN-junction (diode).