ECE241 – Electronics I

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March 1, 2020

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1 Introduction to Electronics

1.1 Basic Concepts

distortion when a signal's output is not a linear function of input (i.e., non-linear transfer function, signal output shape different from input shape)

analog signals difficult to process b/c of noise, distortion, difficult to store because require an analog storage method (e.g., capacitors)

many digital signals still have to be processed as analog signals (e.g., when retrieved from storage) before becoming a robust digital signal, as analog signals are the raw form and digital signals are an abstraction

analog functions include amplification (characterized by gain, speed, power distribution, (practical) bandwidth), filtering

KCL related to conservation of charge; KVL related to conservation of energy

2 Semiconductor Physics

2.1 Basic Concepts

band gap an energy range in which electrons cannot exist (i.e., between valence shell and conduction layer)

band gap energy the difference between top and bottom of band gap energy; i.e., the energy required to move an electron into the conduction layer from the valence shell

2.2 Transport of Carriers

drift movement of charge carriers due to an E-field; since E and q are both signed, both equations (for holes and electrons) are positive; drift current proportional to E and Anq (charge density per unit cross-sectional area) with proportionality constant μ_n (mobility)

diffusion movement of charge carriers due to an uneven charge distribution; for positive charges/current, moves in opposite direction of gradient, so sign is negative (positive for electrons b/c of sign of q); proportional to q, $\frac{dn}{dx}$, with proportionality constant D_n (diffusion constant)

3 pn Junction

3.1 pn Junction in Equilibrium

diffusion current diffusion current from p to n side

depletion region ions without free charges build up in the center region

E-field E-field from n to p side

equilibrium constraints on currents $|I_{dr,p}| = |I_{di,p}|$, $|I_{dr,n}| = |I_{di,n}|$

4 Carrier modeling in equilibrium

band gap The difference in energy between the conduction band and the valence band. In metals, it may be zero; in insulators, it is usually high; in semiconductors, it is small.

5 Symbols, Equations, and Units

electron volt (eV) 1eV = 1.6×10^{-19}J (i.e., since $V = \frac{J}{C}$, $1V = \frac{1eV}{1e}$, and $e = 1.6 \times 10^{-19} C$) Brief review of physics because dumb:

$$\vec{F} = q\vec{E}$$

$$\int \vec{F} \cdot d\vec{s} = W,\ \vec{F} = -\nabla U$$

$$\int \vec{E} \cdot d\vec{s} = V,\ \vec{E} = -\nabla V$$

$$-U = W = -qV$$

(Voltage is EPE per unit charge, electric field is electric force per unit charge. Integrate force to get energy, or field to get potential. The electric potential is the negative of the work done by the field; electrical potential energy and electric potential have the same sign.)

band gap energy (E_g) for Si @ room temp., $E_g = 1.12\text{eV}$

mass action law $np = n_i^2$

intrinsic electron density (n_i) for Si, $n_i = 5.2 \times 10^{15} \exp \frac{-E_g}{2kT} \frac{e^-}{\text{cm}^3}$; in intrinsic material, $n \approx n_i \approx p$

negative charge carrier density (n) n-type: $n \approx N_D$; p-type: $n \approx \frac{n_i^2}{N_A}$

positive charge carrier density (p) p-type: $p \approx N_A$; n-type: $n \approx \frac{n_i^2}{N_D}$

current density $I = JA$

drift current density $J_{dr} = q(\mu_n n + \mu_p p)E$

mobility saturation $\mu(E) = \frac{\mu_0}{1+bE} \Rightarrow v \approx \mu_0 E$ at low E, $v \approx \frac{\mu_0}{b}$ at high E

diffusion current density $J_{di} = q\left(D_n \frac{dn}{dx} - D_p \frac{dp}{dx}\right)$

Einstein relation $\frac{D}{\mu} = \frac{kT}{q}$; at 300K, $\frac{kT}{q} \approx 26mV$

anode where current flows into device; ACID (Anode Current Into Device); for batteries negative terminal; for passive devices positive terminal

built-in potential of a diode $V_0 = \frac{D_p}{\mu_p} \ln \frac{p_p}{p_n} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$

effective mass $\vec{F} = -q\vec{E} = m_0 \frac{dv}{dt}$ (but in a whole semiconductor crystal, use define masses m_n^* and m_p^*); allows treatment of carriers as quasi-classical particles

state densities just above the conduction band, the density of electrons is

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3}$$

and similarly below the valence band for the density of holes:

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3}$$

These are symmetric, albeit with a different constant.

Fermi function The Fermi function specifies how many of the existing states at the energy E will be filled with an electron:

$$f(E) = \frac{1}{1 + \exp\frac{E - E_F}{kT}}$$

$\lim_{T\to 0} f(E)$ is a the step function $1 - H(E_F)$. If T is a positive finite number, then it approaches a smooth curve with $f(F_E) = 1/2$. Increasing temperature makes the slope shallower (i.e., widening out the probability curve). The function is rotationally symmetric about the point $(F_E, 1/2)$; thus if the probability that a conduction state is full is equal to the probability that a valence state is empty, then $E_F = \frac{E_C + E_V}{2} = E_I$ (the midgap). We can make the approximations:

- If $E \geq E_F + 3kT$ (if E sufficiently higher than the Fermi level), then $f(E) \approx \exp -\frac{E - E_F}{kT}$ (the 1 becomes insignificant, and it acts like a negative exponential)
- If $E \leq E_F - 3kT$, then $f(E) \approx 1 - \exp\frac{E - E_F}{kT}$ (thus, as E decreases, then $1 - f(E) = \exp\frac{E - E_F}{kT}$ drops off exponentially, acting like a negative exponential)
- At room temperature, $3kT = 0.0777\text{eV}$ is quite small, so this approximation can often be used.

To summarize, approximations:

$$f(E) \approx \exp -\frac{E - E_F}{kT},\ E \geq E_F + 3kT$$

$$1 - f(E) \approx \exp\frac{E - E_F}{kT},\ E \leq E_F - 3kT$$

Note that the former models electron density in the conduction band, and the latter models hole density in the valence band. Note that, visually, the number of the negative carriers will be greater than the number of positive carriers if $E_C - E_F \leq E_F - E_V$, and vice versa; i.e., the band the Fermi level is closer to will have a higher density of charge carriers.

carrier densities The product of the state densities and Fermi probability function at any energy level gives the number of carriers at that energy level. Integrating, we get

$$n = \int_{E_C}^{E_{top}} g_c(E) f(E)\, dE$$

$$p = \int_{E_{bottom}}^{E_V} g_v(E)(1 - f(E))\, dE$$

To simplify this, we define the Fermi-Dirac integral of order 1/2, and other substitutions:

$$F_{1/2}(\eta_x) = \int_0^\infty \frac{\eta^{1/2}\, d\eta}{1 + e^{\eta - \eta_x}}$$

$$\eta = \frac{E - E_c}{kT},\ \eta_c = \frac{E_F - E_c}{kT},\ \eta_v = \frac{E_v - E_F}{kT}$$

$$N_C = 2\left(\frac{m_n^* kT}{2\pi\hbar^2}\right)^{3/2},\ N_V = 2\left(\frac{m_p^* kT}{2\pi\hbar^2}\right)^{3/2}$$

then

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c)$$

$$p = N_V \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_v)$$

These are the general result (since this comes from an integral of the product of the unapproximated Fermi integral). Thus we use the same approximations when the semiconductor is nondegenerate (i.e., when $E_v + 3kT \leq E_F \leq E_c - 3kT$)