ECE211 – PSET 3

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1. Convolution.

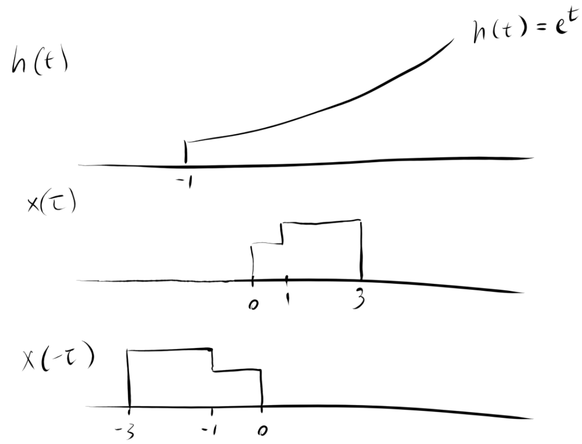
   Let

$$h(t) = e^t u(t+1)$$

$$x(t) = \begin{cases} 2 & 0 \le t \le 1 \\ 3 & 1 \le t \le 3 \\ 0 & \text{else} \end{cases}$$

   and

$$y = h * x$$



   There are four areas with different formulas here due to the piecewise nature of both functions

   (a) The first region is when $t \ge 2$, and the entire mirrored x region overlaps with the exponential where it is positive ($t \ge 1$). The integral with the proper bounds is

$$y(t) = \int_{t-3}^{t-1} 3e^{\tau}\, d\tau + \int_{t-1}^{t} 2e^{\tau}\, d\tau = 2e^t + e^{t-1} - 3e^{t-3},\ t \ge 2$$

(b) The second region is when $t \geq 2$, when the entire region where $x = 2$ overlaps with the positive exponential, but the overlap between the region where $x = 3$ and the exponential is only in the region $-1 \leq t \leq t-1$. Thus the convolution here is

$$y(t) = \int_{-1}^{t-1} 3e^{\tau}\, d\tau + \int_{t-1}^{t} 2e^{\tau}\, d\tau = 2e^t + e^{t-1} - 3e^{-1},\ 0 \leq t \leq 2$$

(c) The third region is when none of the region where $x = 3$ overlaps with the positive exponential, and some of the region where $x = 2$ overlaps with the positive exponential.

$$y(t) = \int_{-1}^{t} 2e^{\tau}\, d\tau = 2(e^t - e^{-1}),\ -1 \leq t \leq 0$$

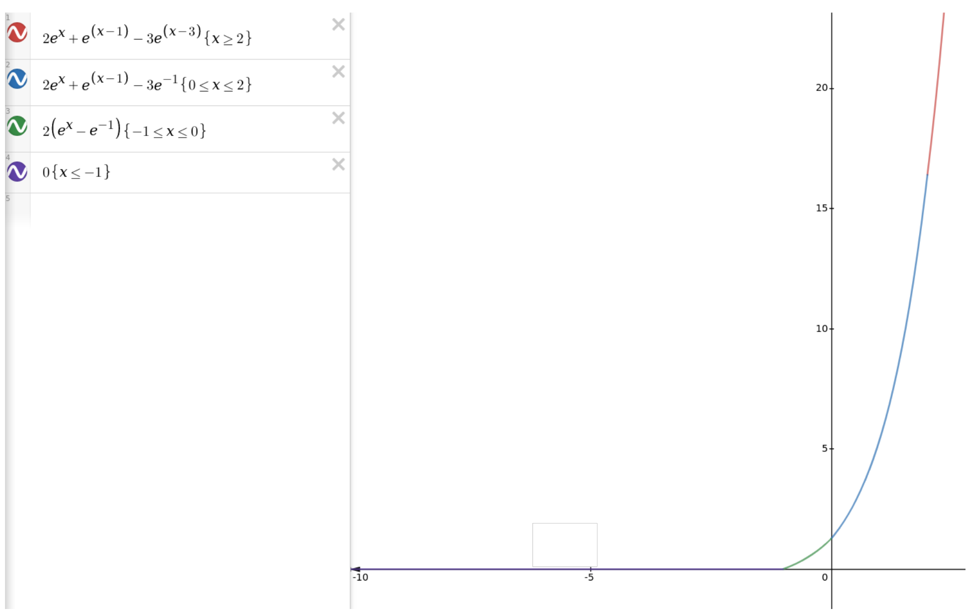
(d) The last region is where there is no overlap. Thus the convolution is zero.

$$y(t) = 0,\ t \leq -1$$

Then

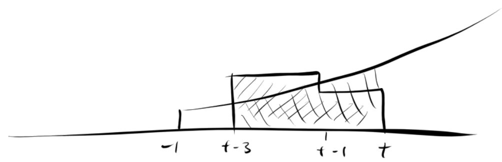
$$y(t) = \begin{cases} 2e^t + e^{t-1} - 3e^{t-3}, & t > 2 \\ 2e^t + e^{t-1} - 3e^{-1}, & 0 < t \leq 2 \\ 2(e^t - e^{-1}), & -1 < t \leq 0 \\ 0, & t \leq -1 \end{cases}$$

The bounds here are chosen arbitrarily, since $y(x)$ is a continuous function, as can be clearly seen in the following figure.



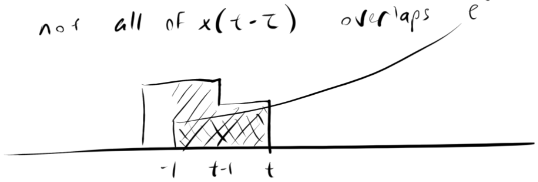
Case 1): $t \geq 2$

all of $x(t-\tau)$ overlaps with e^{t}



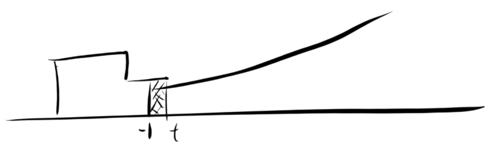
Case 2): $0 \leq t \leq 2$

not all of $x(t-\tau)$ overlaps e^{t}



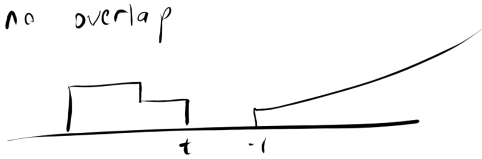
Case 3): $-1 \leq t \leq 0$

not all of $x(t-\tau)$ overlaps e^{t}



Case 4): $t \leq -1$

no overlap



2. Let $h = \{5, \underline{3}, 2, -2, -3\}$, $x = \{2, 3, \underline{4}, 5, 6\}$, $y = h * x$.

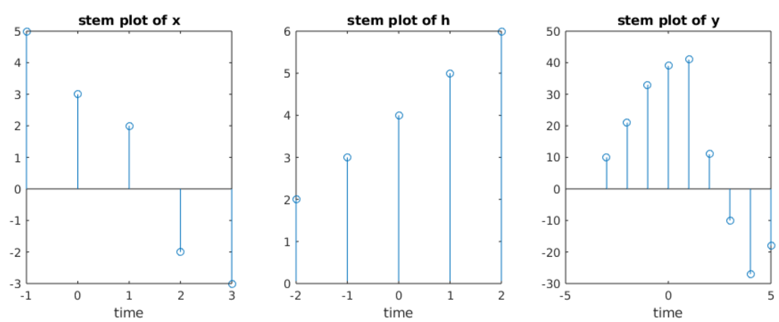
(a)

	Length	Support
x	5	$[-2, 2]$
h	5	$[-1, 3]$
y	9	$[-3, 5]$

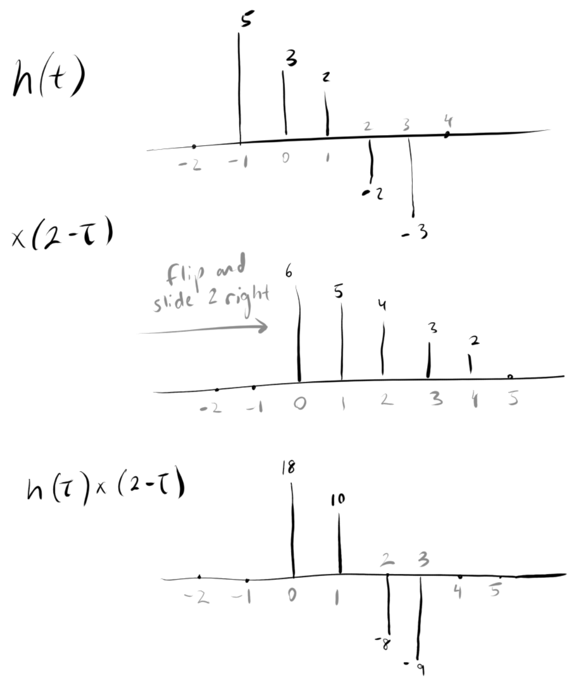
(b)

```
% generate signals
x = [5 3 2 -2 -3];
x_start = -1;
h = [2 3 4 5 6];
h_start = -2;
```

```
y = conv(h, x);
y_start = x_start + h_start;

% finite discrete stem
fds = @(s, s_start)
        stem((0:length(s)-1)+s_start , s);

% plot signals
subplot(1,3,1);
fds(x, x_start);
title('stem plot of x');
xlabel('time');

subplot(1,3,2);
fds(h, h_start);
title('stem plot of h');
xlabel('time');

subplot(1,3,3);
fds(y, y_start);
title('stem plot of y');
xlabel('time');
```

(c) 

(d) Flip 'n slide:



$$y[2] = \cdots + h[-3]x[5] + h[-2]x[4] + h[-1]x[3] + h[0]x[2] + h[1]x[1]$$

$$+h[2]x[0] + h[3]x[-1] + h[4]x[-2] + h[5]x[-3]\ldots$$

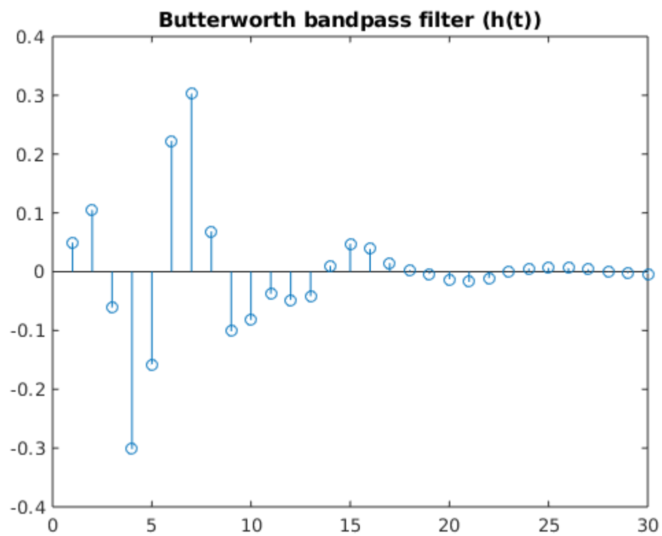
$$= \cdots + 0\cdot 0 + 5\cdot 0 + 3\cdot 6 + 2\cdot 5 + (-2)\cdot 4 + (-3)\cdot 3 + 2\cdot 0 + 0\cdot 0 + \cdots = 11$$

3. The Butterworth bandpass filter

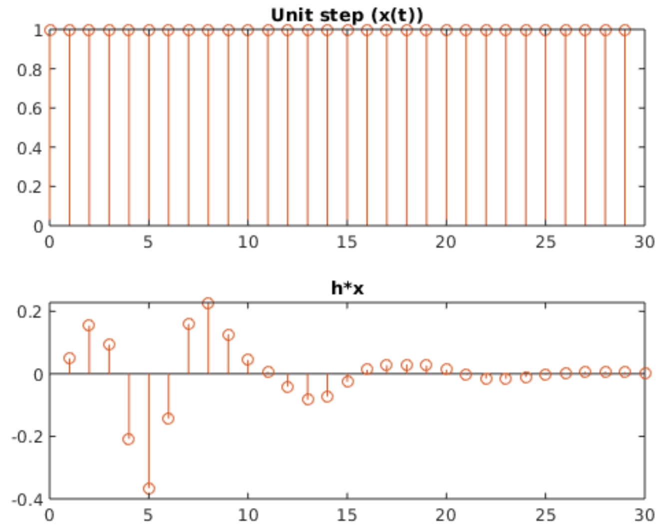
```
[b, a] = butter(3, [0.2 0.5]);
[h, t] = impz(b, a, 30);

figure;
stem(h);
title('Butterworth bandpass filter (h(t))');

figure;
x = ones(30);
subplot(2,1,1);
stem(t, x);
title('Unit step (x(t))');
subplot(2,1,2);
stem(filter(b, a, x));
title('h*x');

figure;
```

```
t = 0:30;
x = cos(0.4*pi*t);
subplot(2,1,1);
stem(t, x);
title('cos(4*pi*t) (x(t))');
subplot(2,1,2);
stem(filter(b, a, x));
title('h*x');
```

(a) Butterworth response with passband from $0.2 \leq \omega_{pass} \leq 0.5\,\text{rad}$



(b) Intially, the output looks like the impulse response $h(t)$, since giving it a starting value of 1 acts like an impulse. However, since $\omega = 0\text{rad}$ is out of the passband, the output quickly dies out to almost no amplitude.



(c) Since $\omega = 0.2\text{rad}$ is in the passband, the signal is not noticeably attenuated at all and matches the input at all of the sample points.