Trig identities

Jonathan Lam

February 15, 2020

1 Basic properties

$$\sin -\theta = -\sin\theta$$
$$\cos -\theta = \cos\theta$$
$$\sin^2\theta + \cos^2\theta = 1$$

2 Angle sum identities

Note Euler's identity:

$$e^{i\theta} = cos\theta + i\sin\theta$$
$$\Rightarrow e^{i(\theta_1+\theta_2)} = e^{i\theta_1}e^{i\theta_2} = (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$$
$$= (\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1)$$
$$= \cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)$$

3 Sine/cosine square and identities

Derive using $\cos 2\theta$ identities, i.e.,

$$\cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

Useful identities:

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$
$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$$

4 Sine/cosine multiplication properties

Derive using angle sum formulas, e.g.,

$$\cos(\theta_1 + \theta_2) = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$$

$$\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$$

$$\Rightarrow \cos(\theta_1 + \theta_2) + \cos(\theta_1 - \theta_2) = 2\cos\theta_1 \cos\theta_2,$$

$$\cos(\theta_1 + \theta_2) - \cos(\theta_1 - \theta_2) = -2\sin\theta_1 \sin\theta_2$$

Useful identities:

$$\cos\theta_1 \cos\theta_2 = \frac{\cos(\theta_1 + \theta_2) + \cos(\theta_1 - \theta_2)}{2}$$

$$\sin\theta_1 \sin\theta_2 = \frac{\cos(\theta_1 - \theta_2) - \cos(\theta_1 + \theta_2)}{2}$$

$$\sin\theta_1 \cos\theta_2 = \frac{\sin(\theta_1 + \theta_2) + \sin(\theta_1 - \theta_2)}{2}$$

(Note that $\cos\theta_1 \sin\theta_2 = \sin\theta_2 \cos\theta_1$.)

5 Sine/cosine sum properties

Derive using sine/cosine multiplication properties, e.g.,

$$\frac{\cos\theta_1 + \cos\theta_2}{2} = \cos\theta_1' \cos\theta_2'$$

$$\theta_1 = \theta_1' + \theta_2',\ \theta_2 = \theta_1' - \theta_2' \Rightarrow \theta_1' = \frac{\theta_1 + \theta_2}{2},\ \theta_2' = \frac{\theta_1 - \theta_2}{2}$$

Useful identities:

$$\cos\theta_1 + \cos\theta_2 = 2\cos\frac{\theta_1 + \theta_2}{2} \cos\frac{\theta_1 - \theta_2}{2}$$

$$\cos\theta_1 - \cos\theta_2 = -2\sin\frac{\theta_1 + \theta_2}{2} \sin\frac{\theta_1 - \theta_2}{2}$$

$$\sin\theta_1 + \sin\theta_2 = 2\sin\frac{\theta_1 + \theta_2}{2} \cos\frac{\theta_1 - \theta_2}{2}$$

(Note that $\sin\theta_1 - \sin\theta_2 = \sin\theta_1 + \sin -\theta_2$.)