Solve the first-order linear ODE

$$y' + P(x)y = Q(x)$$

Assume that there exists a function $\mu(x)$ s.t. $\mu(x)(y' + P(x)y) = (\mu(x)y)'$. Then

$$(\mu(x)y)' = \mu(x)Q(x)$$

$$y = \mu^{-1}(x) \int \mu(x)Q(x)\,dx$$

To find $\mu(x)$ (the **integrating factor**), expand the condition used to define it by product rule, i.e.,

$$\mu'(x)y + \mu(x)y' = \mu(x)y' + \mu(x)P(x)y$$

This is a separable first-order ODE.

$$\mu'(x) = \mu(x)P(x)$$

$$\frac{d(\mu(x))}{\mu(x)} = P(x)dx$$

$$\ln|\mu(x)| = \int P(x) + C$$

$$\mu(x) = C \exp \int P(x)\,dx$$

Note that since there exists a $\mu^{-1}(x)$ and a $\mu(x)$ in the solution (in the integral), the constant multiplier will cancel out, and therefore we can disregard it when calculating the integrating factor. In summary,

$$y = e^{-\int P(x)\,dx} \left(\int e^{\int P(x)\,dx} Q(x)\,dx\,(+C) \right)$$