Basic DC Circuit Analysis Lab

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I. Introduction

The objective of this lab is to become familiar with the process of analyzing simple DC circuits. It is an introductory lesson to the theoretical and practical sides of using two basic circuit elements, resistors and power sources, and two pieces of equipment, power supplies and digital multimeters (DMMs). The lab will begin with a review of resistor color bands, and then requires the use of Ohm's Law, the voltage divider rule, and the current divider rule to solve for V, I, and R in basic circuits. An understanding of KVL and KCL are also reinforced with some theoretical questions. The practical side is to build these circuits, to measure electrical aspects of the circuits, and to calculate other measures (and the percent errors) to determine how closely the theoretical laws hold true in practice.

II. Theory and Prelab

The following calculations were done in preparation for the lab.

Note: For the calculations in this theory section, a bar $\bar{x}$ indicates the last significant digit if more digits are shown than are significant. Only final results are rounded to the correct number of significant digits; intermediate steps may be shown rounded for concision.

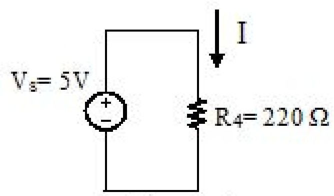
Resistor review

A review of resistor color bands and interpretations is shown in the below table.

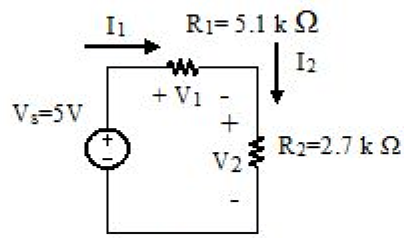
Table 1. Resistor colors and values

Color of band 1	Color of band 2	Color of band 3	Color of band 4	Resistor value
blue	red	red	silver	6200 ± 620 Ω
red	violet	brown	gold	270 ± 14 Ω
yellow	orange	black	gold	43 ± 22 Ω
brown	gray	green	gold	1800 ± 90k Ω
red	violet	orange	gold	27000 ± 1350 Ω
white	brown	brown	gold	910 ± 46 Ω
red	red	orange	gold	22 ± 1.1k Ω

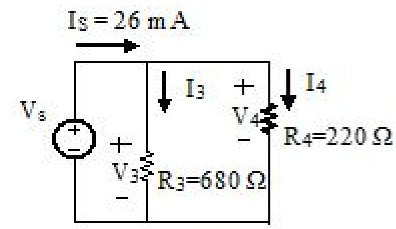
Circuit diagrams



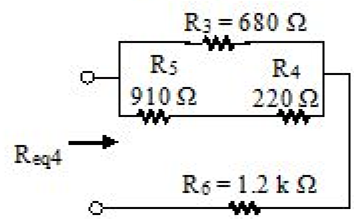
Circuit 1



Circuit 2



Circuit 3



Circuit 4

Circuit value calculations

These values were calculated to estimate results for the lab. They appear in the “pre-lab” section of the tables in Section IV: Results and Discussion, and use the desired resistor values from the circuit diagrams (which may differ slightly from the actual resistor values).

<u>1. Ohm’s law</u>

Solve for I in Circuit 1:

$I = \frac{V}{R} = \frac{5V}{220\Omega} = \bar{2}3mA$

<u>2. KVL</u>

Determine V_1 and V_2 for circuit 2, using the voltage divider rule.

$V_1 = 5V \times \frac{5.1k\Omega}{5.1k\Omega+2.7k\Omega} = \bar{3}.27V$

$V_2 = 5V \times \frac{2.7k\Omega}{5.1k\Omega+2.7k\Omega} = \bar{1}.73V$

Write a KVL equation for the circuit.

$-V_s + V_1 + V_2 = 0V$

$-5V + 3.27V + 1.73V = 0.V$

Calculate the currents I_1 and I_2 using Ohm’s Law and the voltages just computed

$I_1 = \frac{\bar{3}.27V}{5.1k\Omega} = 0.\bar{6}4mA$

$I_2 = \frac{\bar{1}.73V}{2.7k\Omega} = 0.\bar{6}4mA$

Their currents are the same, which makes sense because they form part of the same loop (in series).

3. KCL

Determine I_3 and I_4 for Circuit 3, using the current divider rule.

$I_3 = 26mA \times \frac{1/680\Omega}{1/680\Omega + 1/220\Omega} = 6.\bar{3}6mA$

$I_4 = 26mA \times \frac{1/220\Omega}{1/680\Omega + 1/220\Omega} = 1\bar{9}.6mA$

Calculate the voltages V_3 and V_4 using Ohm's law and the currents just computed.

$V_3 = IR = 6.\bar{3}6mA \times 680\Omega = 4.3V$

$V_4 = IR = 1\bar{9}.6mA \times 220\Omega = 4.3V$

The voltages are the same because they connect the same two nodes (i.e., act in parallel)

Write a KCL equation for the circuit.

$-I_s + I_3 + I_2 = 0$

$-26mA + 6.\bar{3}6mA + 1\bar{9}.6mA = 0.mA$

Determine equivalent resistance seen by the voltage source for circuit 3 and then determine the value of the voltage source V_s needed to provide a supply current $I_s = 26mA$.

$R_{eq3} = \frac{R_3 R_4}{R_3 + R_4} = \frac{680\Omega \times 220\Omega}{680\Omega + 220\Omega} = 1\bar{6}6\Omega$

$V_s = IR = 0.026A \times 166\Omega = 4.3V$

4. Series and parallel resistances

Determine the equivalent resistance, R_{eq4}, for circuit 4.

$R_{eq4} = 1.2\Omega + \frac{1}{1/680\Omega + 1/(910\Omega + 220\Omega)} = 1\bar{6}24\Omega$

III. Test procedure, equipment, and data

The test setup involved a tabletop digital multimeter, an adjustable power supply, and carbon resistors. All of the recorded measured values were measured with the multimeter, and all digits provided by the multimeter display are recorded.

The desired and measured resistor values are shown in Table 2.

Table 2. Desired and actual resistor values

	Desired resistance	**Components used** (gold tolerances)	**Measured resistance**
R_1	5.1kΩ	5.1kΩ	5.0620kΩ
R_2	2.7kΩ	2.2kΩ + 510Ω	2.6653kΩ
R_3	680Ω	560Ω + 100Ω	654.41Ω
R_4	220Ω	220Ω	218.08Ω
R_5	910Ω	910Ω	902.58Ω
R_6	1.2kΩ	1.2kΩ	1.17837kΩ

The voltage source voltage is shown in Table 3.

Table 3. Desired and actual voltage source voltage

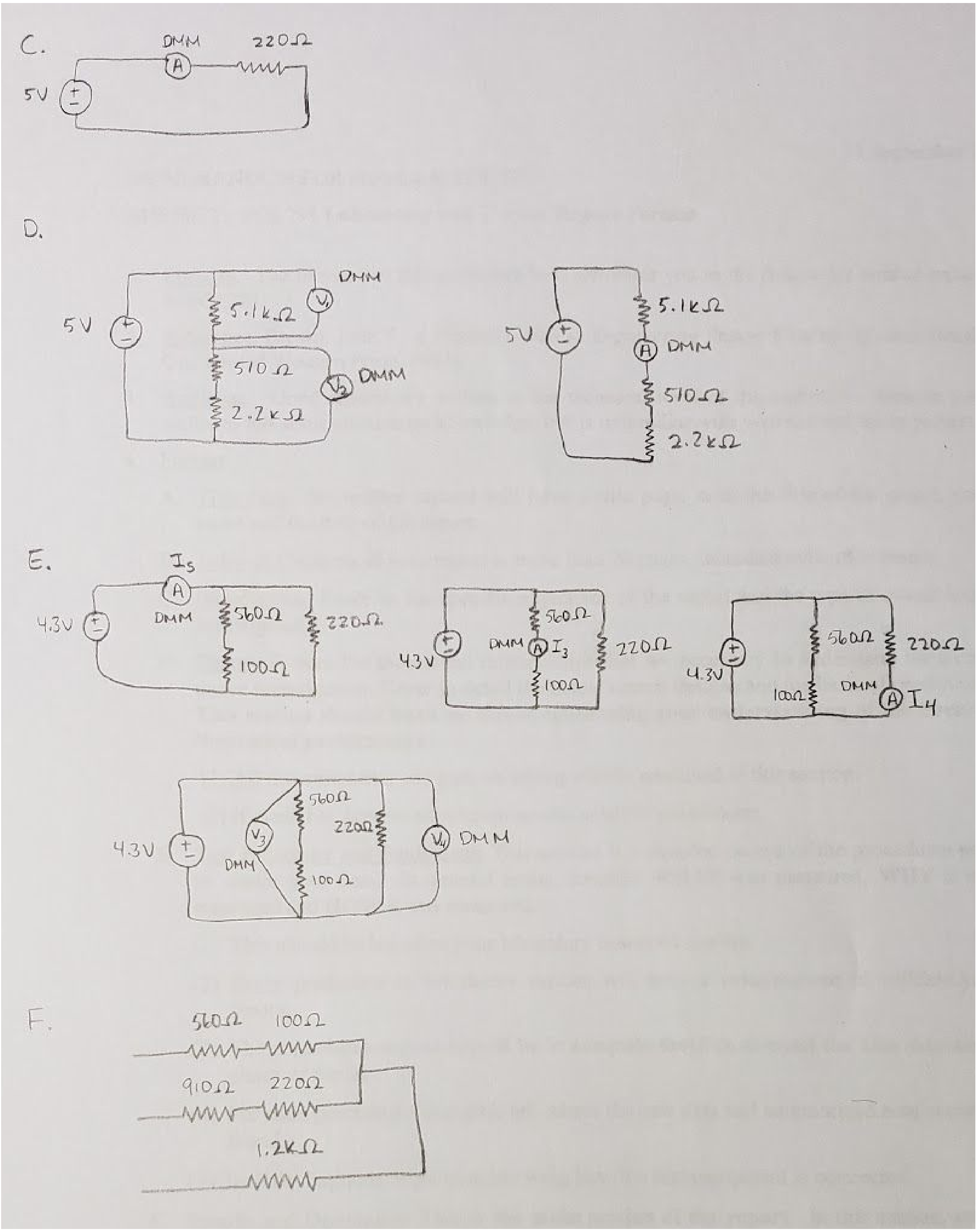
	Desired	**Measured**
$\mathbf{V_s}$	5V	4.96025V

The experimental setups for measuring current and voltage with the multimeter are shown in Figure 1. The measured current and voltage values are shown in Table 4.

Table 4. Measured voltages, currents, and resistances

		Measured value
Circuit 1	I	22.600mA
Circuit 2	V_1	3.2482V
	V_2	1.711V
	I_1	0.64128mA
	I_2	0.64125mA
Circuit 3	I_s	25.635mA
	I_3	6.4335mA
	I_4	19.2585mA
	V_3	4.2195V
	V_4	4.2193V
Circuit 4	R_{eq}	$1.59199k\Omega$

Figure 1. Equipment setup



Results and Discussion

Note: In the following sections, the percent error formula is
$\%\ err = \frac{|experimental - theoretical|}{theoretical} \times 100\%$, and the calculation will be omitted.

Part A. Ohm's Law

The theoretical current and voltage are calculated with the following calculations using measured values.

$I = \frac{V_s}{R_4} = \frac{4.96025V}{218.08\Omega} = 22.745mA$

$V = I_{obs}R = 22.600mA \times 218.08\Omega = 4.9286V$

Table 5. Circuit 1 current and voltage values

Metric	Prelab	Theoretical	Experimental	Percent Error
I	$\bar{2}3mA$	22.745mA	22.600mA	0.638%
V	5V	4.9286V	4.96025V	0.638%

This result verifies Ohm's law, since it is used to calculate current given voltage and resistance.

Part B. KVL and voltage divider rule

The theoretical values for V_1, V_2 I_1, and I_2 are calculated for circuit 2 as follows using measured values:

$V_1 = 4.96025V \times \frac{5.0620k\Omega}{5.0620k\Omega + 2.6653k\Omega} = 3.2493V$ (voltage divider)

$V_2 = 4.96025V \times \frac{2.6653k\Omega}{5.0620k\Omega + 2.6653k\Omega} = 1.7109V$ (voltage divider)

$I_1 = \frac{3.2493V}{5.0320k\Omega} = 0.64573mA$

$I_2 = \frac{1.7109V}{2.6653k\Omega} = 0.64191mA$

Table 6. Circuit 2 current and voltage values

Metric	Prelab	Theoretical	Experimental	Percent Error
V_1	$\bar{3}.27V$	3.2493V	3.2482V	0.03385%
V_2	$\bar{1}.73V$	1.7109V	1.7110V	0.005849%
I_1	$0.\bar{6}4mA$	0.64573mA	0.64125mA	0.50021%
I_2	$0.\bar{6}4mA$	0.64191mA	0.64128mA	0.098145%

This result verifies the voltage divider rule, as it was used to calculate the voltage across two resistors in series. As for any circuit, it also demonstrates KCL, since the current across circuit elements in series are equal, and KVL, because the sum of the voltage drops of the resistors is equal to the voltage source potential.

Voltage must be measured *across* a circuit element because it is only defined as a relative energy potential, i.e., with reference to another node. In other words, voltage across a circuit element is only the difference in potential between the two ends of the circuit element.

Part C. KCL and current divider rule

Theoretical values for I_3, I_4, V_3, V_4, and i_s for Circuit 3 were calculated as follows using measured values:

$I_3 = 25.635mA \times \frac{1/654.41\Omega}{1/654.41\Omega + 1/218.08\Omega} = 6.4075mA$ (current divider)

$I_4 = 25.635mA \times \frac{1/218.08\Omega}{1/654.41\Omega + 1/218.08\Omega} = 19.227mA$ (current divider)

$V_1 = IR = 6.4075mA \times 654.41\Omega = 4.1931V$

$V_2 = IR = 19.227mA \times 218.08\Omega = 4.1930V$

$I_s = I_3 + I_4 = 6.4335mA + 19.2585mA = 25.6920$

Table 7. Circuit 3 current and voltage values

Metric	Prelab	Theoretical	Experimental	Percent Error
I_3	$6.\bar{3}6mA$	6.4075mA	6.4335mA	0.40577%
I_4	$1\bar{9}.6mA$	19.227mA	19.2585mA	0.1638%
V_1	4.3V	4.1931V	4.2195V	0.62961%
V_2	4.3V	4.1930V	4.2193V	0.69878%
I_s	26mA	25.6920mA	25.6350mA	0.22185%

This circuit analysis directly illustrates the use of the current divider rule, as it is used to calculate the current across multiple branches given the (inverse) resistances of each branch. It also demonstrates KCL, as the sum of the currents through the two branches is equal to the current entering the branches (I_s). As with Parts A and B, the percent errors are very small here, for likely the same reasons as before.

Here, as opposed to voltage, current must be measured by “breaking” the circuit because the current is calculated at a node, rather than as a difference between nodes. An ammeter can only measure the current at a node by having the node’s current pass through it, thus requiring “breaking” the circuit.

Part D. Equivalent Resistance

The theoretical equivalent resistance calculation is shown below using measured values, and a summary of the theoretical and measured values is shown in Table 7 below.

$$R_{eq4} = 1.17837k\Omega + \frac{1}{1/654.41\Omega + 1/(902.58\Omega + 218.08\Omega)} = 1591.52k\Omega$$

Table 8. Circuit 4 equivalent resistance

Metric	Prelab	Theoretical	Experimental	Percent Error
R_{eq}	1.624k Ω	$1.59152k\Omega$	$1.59199k\Omega$	0.029531%

The calculation uses the fact that the equivalent resistance of resistors in series is the sum of the resistances (a fact that can be derived from KCL), and the fact that the equivalent resistance of resistors in parallel is the inverse sum of the inverses of the resistances (a fact that can be derived from KVL).

The power source must be removed when measuring resistance because the multimeter's method of testing resistance is to apply a voltage, measure the current, and calculate resistance using Ohm's law from these two known values. Applying an external voltage across the circuit adjusts this voltage, and thus the Ohm's law calculation is wrong.

Resistance is symmetric with probe polarity, since resistance is a physical property of a resistor and therefore independent of voltage or current passing through it. Voltage, however, is a directed difference between two potentials, so swapping probe polarities naturally reverses the sign of the difference. Similarly, current is a directed flow of electrons, so swapping probe polarities also reverses the sign of current.

Error analysis

The percent error of all of the theoretical calculations is very small. All of the percent errors were less than 0.6%, and most were within 0.01%, which all fall within the acceptable tolerance of within 3%.

The small error is likely to be due mostly to the measured resistance being incorrect, since the value was not extremely stable over time (a known physical property of resistors as they heat up), a modeling error with the assumption that the resistance of a resistor is constant. Another possibility in error is that voltage and resistance (or voltage and current) can be slightly nonlinear, a modeling error with Ohm's law. Again, both of these modeling errors are expected to be very small for practical applications, and the resulting error is therefore not large.

The error is less likely to be largely impacted by measurement (instrumental) error, since the instrumental precision is very high — from the measurements, the multimeter gave readings to the precision of ±100nA, ±10mΩ, and ±1000μV.

There should be no parametric error in the measured values from the lab, since the resistor and voltage source values were also measured (and therefore those measured values, and not standard values, were used in the calculations).

IV. Conclusions

Firstly, this lab reviews resistor band color-codes. This lab reinforces the fundamental relations between some basic measures of DC circuits: resistance, voltage, and current. Ohm's law creates a simple relation for a resistive load; for more complex circuits in series or parallel, the understanding of KVL (zero-sum voltage over a loop) and KCL (zero-sum current through a point) allows for the derivation and use of the voltage and current divider laws. Lastly, it is a good review on how to practically measure voltage and current in a circuit using a multimeter, and shows that the deviation of empirical values from from the theoretical values is very small (by the small percent errors), which means that the theory is a sound basis for prediction of resistance, voltage, and current in basic DC circuits.

V. Recommendations

This lab was useful in showing the basics of circuit analysis. While many of the sophomore electrical engineers had taken DLD, this is the first class in which the analysis method is paired with the practical side, so it does serve a useful purpose in the class.

A critique is that the lab report given to the students may be a little over-structured. It may be more educational if the students were told to solve for the same values in the circuits without explicitly mentioning what method to use (e.g., voltage and current divider laws). Or, perhaps it would be better to ask students to solve for a value multiple ways (e.g., using Ohm's law and the voltage divider law).

VI. References

Shay, Lisa A. "ECE 291 Laboratory and Project Report Format." *The Cooper Union*, 11 Sept. 2019.