PHYSICS 112 MIDTERM REVIEW
Wolf Sections A & C, Spring 2019

Document Metanotes:
- The sections are topics and may not correspond to a chapter
- Using 8th edition Halliday & Resnick (H&R) problem numbers
- For all equations, don't have to memorize them, but know how to derive quickly
- LaTeX generated using plugin Auto-LaTeX Equations
- Any errors were unintentional

Capital T Truths
- $\sum \vec{F}_{net} = m\vec{a}$
- Newton's third law
- Conservation of total energy
  - Sometimes conservation of mechanical energy
- Conservation of linear momentum

General Problem-Solving Skills
- Draw FBDs for each system/object and extra diagrams as necessary
- Write Newton's second law for each system/object
- Label systems/objects (and, for conservation of energy and linear momentum, also label start/end times)
- Explain in words when necessary
- Don't do extra algebra unless necessary
- Box important equations
- (Quickly) take limits to check answers

Chapter 3: Vectors
- Understand vector operations (addition, subtraction, dot, cross)
  - Dot product is "a measure of parallelness," involved with projections; max when vectors parallel, 0 when orthogonal
  - Cross product is "a measure of perpendicularity"; max when vectors orthogonal, 0 when parallel
    - Know right-hand rule
- Know how to convert between Cartesian and magnitude-angle notation
- Understand how to break up vectors into its parts
  - Especially for ramp questions: understand how to break up vectors parallel/perpendicular to incline into vectors in the x/y directions and vv
- Understand rotation of a coordinate axes by some angle (analogous to translation by some vector)
- Understand spherical coordinates ($P(\rho, \theta, \phi)$)
- $\vec{x}$, $\vec{v}$, $\vec{t}$, $\vec{F}$, and $\vec{p}$ are all vector quantities (i.e., Newton's second law is a vector quantity, and write it so unless analyzing motion in 1D)
- Significant problems:
  - Ramps
  - Torque and significance of direction of torque vector

Chapters 2 & 4: One- and Two-Dimensional (Constant Acceleration) Kinematics
- $\vec{v} = \frac{d\vec{x}}{dt}$, speed $= |\vec{v}|$, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$

- (Don't get confused between speed and velocity! Speed is rate of change in distance, velocity is rate of change of displacement)
- Five basic constant-a kinematics eqs:
  - $v = v_0 + at$ $(a,\, t,\, v_0,\, v)$
  - $\Delta x = \frac{1}{2}at^2 + v_o t$ $(\Delta x,\, a,\, t,\, v_0)$
  - $\Delta x = -\frac{1}{2}at^2 + vt$ $(\Delta x,\, a,\, t,\, v)$
  - $\Delta x = \frac{1}{2}(v_0 + v)t$ $(\Delta x,\, t,\, v_0,\, v)$
  - $v^2 - v_0^2 = 2a\Delta x$ $(\Delta x,\, a,\, v_0,\, v)$
  - Know how to derive each equation, and make sure to choose correct equation for the question at hand
- If acceleration is not constant, integrate to get v and/or x (don't use above equations!)
- Know how to interpret a-t, v-t, and x-t graphs for particle's v, a, and x
- Projectile motion:
  - Only net acceleration is gravity acting downwards
  - y-t relation: $\Delta y = -\frac{1}{2}gt^2 + v_0 \sin\theta t$
  - x-t relation: $\Delta x = v_0 \cos\theta$
  - y-x relation: $\Delta y = (-\frac{g}{2(v_0\cos\theta)^2})\Delta x^2 + (\tan\theta)\Delta x$
  - range (to starting height): $\Delta x = \frac{v_0^2 \sin 2\theta}{g}$
  - maximum height: $\Delta y = \frac{v_0^2}{2g}$
- Solution setup:
  - Is it a one-dimensional (displacement vs. time) or two dimensional (y vs. x) question?
  - Is it a constant acceleration kinematics question?
    - Determine which 4 variables are relevant (typically 3 known, 1 known) for kinematics problems, and choose the right equation
    - Would using the y-x parabolic equation be easier than 1D kinematics questions?
- Significant problems:
  - Section A Quiz 1: Incline problem H&R Ch4 #43
  - Section C Quiz 1
  - Train problem: H&R Ch2 #42
  - Stair Problem

<u>Chapter 4: Uniform Circular Motion (The Centripetal Force)</u>

- Centripetal acceleration: $a = \frac{v^2}{r}$ (understand both derivations)
  - This equation also applies to instantaneous acceleration for non-circular (i.e., elliptical) orbits
- Period: $T = \frac{2\pi r}{v}$
- The centripetal force is not a fundamental force, but rather the sum of fundamental forces pointing towards the center of rotation that cause an object to move in uniform circular motion (around a circle at constant velocity)
- Centripetal acceleration/force always aimed directly inwards, velocity always aimed tangentially to circle (perpendicular to acceleration)
- Significant problems:
  - Section C Quiz 2: H&R Ch6 #59

Chapter 4: Relative Motion in Inertial Reference Frames

- Let P refer to some object, A refer to first reference frame, and B refer to second reference frame
- Kinematics (always write subscripts to avoid confusion!):
    - $\vec{x}_{\frac{P}{A}} = \vec{x}_{\frac{B}{A}} + \vec{x}_{\frac{P}{B}}$
    - $\vec{v}_{\frac{P}{A}} = \vec{v}_{\frac{B}{A}} + \vec{v}_{\frac{P}{B}}$
    - $\vec{a}_{\frac{P}{A}} = \vec{a}_{\frac{P}{B}}$
- Significant problems:
    - Boat moving relative to water: Ch4 #82
    - Center of mass reference frames

(Wolf's Class): Relative Motion in Accelerated Reference Frames

- Let P refer to some object, A be a good (inertial) reference frame, and B be a bad (noninertial) reference frame
- You can write Newton's laws *only with a good frame*: $\sum \vec{F}_{net} = ma_{\frac{P}{A}}$
- Because $\vec{a}_{\frac{P}{A}} = \vec{a}_{\frac{B}{A}} + \vec{a}_{\frac{P}{B}}$, $\sum \vec{F}_{net} = m(\vec{a}_{\frac{B}{A}} + \vec{a}_{\frac{P}{B}})$
- Significant Problems:
    - Accelerated Reference Frames Packet (moodle)
    - Quiz 4

Chapters 5 & 6: Newton's Second Law and Forces

- A force is something that causes an object to move; a "push" or "pull"; something that can accelerate mass
    - Thus mass can be thought of as a coefficient of "resistance to accelerate" — the higher the mass, the lower the acceleration with the same force
- Newtonian mechanics breaks down at very high speeds due to special relativity and very small scales due to quantum mechanics
- Newton's Second Law: $\sum \vec{F}_{net} = m\vec{a}$
    - Only works in inertial reference frames
    - The forces considered are only external forces acting *on* the object; internal forces cancel in pairs (Newton's third law)
- Solution Setup:
    - **Always** draw FBDs for each object
    - **Always** write Newton's second law (vectorially) for each relevant object in relevant directions
    - Determine the object/system(s). Define them or circle them if it's not clear from the question
        - **Always** consider one object at a time, and only consider external forces acting on that object.
        - If multiple objects are moving together, you can treat them as one object for simpler problems
    - Choose axes that are easy to work with (e.g., ramp problems: tangential-perpendicular or x-y axes?).
- Significant Problems:
    - (See next section)

Chapters 5 & 6 (& 8): Specific Forces

- Gravity
  - $F_g = -mg$, positive direction is upwards
  - $F_g = -\frac{GMm}{R}$, for distances further from the surface of the Earth (but not closer to the center of the Earth)
- Normal
  - Always comes in pairs
  - Remember that pulleys exert normal force on the object they are attached to
- Tension
  - Pulleys redirect tension of ropes
  - Tension is same throughout a single rope
    - If masses attached tightly to some point between the ends of a rope, then the rope is segmented into two ropes that don't have to have the same tension
  - Rods can either pull or push (either tension or normal force) — in problem setup you can assume either pull or push on both objects, and if sign of solved force is negative, just change your assumption to the other
- Friction
  - If net force on object (on axis of friction) is less than $f_{s_{max}} = \mu_s N$, then static friction keeps it in place and $f_s = \text{net force on axis of friction}$ (i.e., static friction matches net force)
    - Remember that if object not moving, **static friction is usually not equal to** $f_{s_{max}}$ (e.g., if no net force, no static friction)
  - Otherwise, friction is $f_k = \mu_k N$
- Spring (Ch8)
  - $F = -kx$ (Hooke's Law), positive direction is stretched string, x is change in length of spring from equilibrium position
- Solution Setup:
  - "Conservation of Rope Length" principle (for a single rope)
  - Before using friction, determine if static or kinetic (one way is to assume static friction, check necessary frictional force to keep object(s) from sliding, and compare to $f_{s_{max}}$)
  - To determine direction of friction, can try problem first w/o friction ("turn off friction"), or can arbitrarily choose direction of friction and infer direction from sign of solved frictional force
- Significant Problems:
  - Friction problems: will it slide? Or determining static friction
  - Quiz #3: listing all possible scenarios involving friction (don't forget "no movement" is an option!)
  - Pulleys and/or ramps and/or friction
    - Section A Quiz #2: monkey problem Ch5 #57
  - Gravity (exact form) / electromagnetic force between two uniform rods

<u>Chapters 7 & 8: Work, Energy, and Power</u>

- Ch7 interpretation of energy: all energy is energy of movement
  - "Better bookkeeping"
- Ch8 interpretation of energy: there exists potential energy ("energy of configuration")
  - Potential energy of a force F (i.e., gravity, spring PE) is $U_F = -W_F$ (negative work by the force to get it to its "configuration")

- In a potential energy diagram, there must be a total mechanical energy. The difference between the total mechanical energy and the potential energy is the kinetic energy; the kinetic energy must be positive, so the object cannot exist in regions of negative potential energy.
  - $F = -\frac{dU}{dx}$ — **the force at a particular position is the negative of the slope**. This means that particle always has a force directed towards a lower potential energy (think like chemistry, lower potential energy means more stable); can think of the potential energy diagram like hills and the object like a skateboard, which keeps getting "pulled" towards the lowest energy points on the potential energy diagram, oscillating between the bounding turning points

- Energy is always conserved in an isolated system (Law of Conservation of Energy)
  - However, mechanical energy is not always conserved: also thermal energy "lost" and internal energies (e.g., chemical) that are part of the system's entire energy (i.e., $E_{total} = E_{mec} + E_{th} + E_{int}$, $\Delta E_{total_{isolated}} = 0$)
  - Using this fact, you can look at two distinct moments in time and solve for energies if you know E_{total} or E_{mec} is conserved, but be sure to define the start and end time
  - If system is not isolated (if external force pierces the "bubble"), then the amount of work done by the external force is equal to the change in total energy of the system
- Mechanical energy (potential + kinetic energies, not thermal or internal energies) is not always conserved, but if it is conserved problems usually become easier
  - In general: $E_{mec} = U + K$, $\Delta E_{mec} = \Delta U + W$
  - Mechanical energy conserved if no friction, sound, heat loss, or permanent deformation of material (nonconservative forces)
    - Static friction is conservative
    - If kinetic friction present, you can still do calculations with energy with the general conservation of energy formula
  - If E_{mec} conserved: $E_{mec_1} = E_{mec_2}$, $\Delta E_{mec} = 0 \Rightarrow \Delta U = -W \Rightarrow F = -\frac{dU}{dx}$
- Work is force applied over a distance, which corresponds to a change in energy of a system. Different ways to think about work:
  - $W = F \times x$
  - $W = \vec{F} \cdot \vec{x} = \vec{F} \cos(\theta) x$
  - $W = \int \vec{F} \cdot dx$ = area under integral of F-x curve
  - $W = \Delta K$ (Ch7 perspective, all energy is kinetic energy)
  - $W = \Delta E_{total}$ (Ch8 perspective, work done on system is the change in energy of the system (kinetic and potential energies included))
  - $W = -dU$ if mechanical energy is conserved
  - $W_{net} = \sum W$
- Work done by specific forces, and potential energies of those forces:
  - $W_g = -mg\Delta h$, $U_g = mg\Delta h$
  - $W_k = -\frac{1}{2}kx^2$, $U_k = \frac{1}{2}kx^2$
- Kinetic energy is $K = \frac{1}{2}mv^2$
- Power is rate of change of energy
  - $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$ (Ch7, all energy is kinetic energy)
  - $P = \frac{dE}{dt}$
- Solution setup:

- **Always** clearly define (using words is best) the start and end moments of analysis.
- Check: is mechanical energy conserved? I.e., was any work done on the system? Was any heat lost?

- Significant problems:
  - Understanding potential energy diagrams at specific total mechanical energy levels
    - Gravitational potential energy versus radius
    - 6-12 potential energy curve
    - Arbitrary potential energy curves (and quantum tunneling?)
  - Pendulum vs. height: Ch7 #65
  - Work of block pulled at an angle: Ch7 #42
  - Section A Quiz 4: Work to lift spring onto table (variable force, either using integral or COM): Ch8 #32
  - Conveyor belt problem: Ch8 #66, moodle

<u>Chapter 9: Linear Momentum and Center of Mass</u>

- Center of mass (COM): $\bar{x} = \frac{1}{M}\int x\,dm$ (a weighted average of the mass weighted by distance)
  - If constant density: $\bar{x} = \frac{1}{V}\int x\,dV$
  - Choose reference point that makes calculations easy
- If not a point mass or spherically/linearly symmetric, center of mass must be calculated using that formula
  - Center of mass of linearly symmetric object is at line of symmetry
  - Center of mass of spherically symmetric object is at center
  - Because COM calculation is linear, if center of mass of two objects known, COM is the weighted average of those two objects (i.e., can use COM on objects if COMs of entire objects known)
  - Doesn't matter what reference point is chosen, COM is still the same
- Trick: if uniform density object with an air/vacuum bubble, can treat vacuum like negative mass and use the COM formula with the COMs of the object and the bubble
- Can treat object like all external forces acting on center of mass: $\sum \vec{F}_{net} = M\vec{a}_{COM}$
- Linear momentum defined to be $\vec{p} = m\vec{v}$
  - Thus $\sum F_{net} = \frac{d\vec{p}}{dt}$ (another way to write Newton's third law)
  - Thus $\vec{p}_f = \vec{p}_i + \Delta p = \vec{p}_i + \int \vec{F}_{net}dt$
- Impulse is defined as the change in linear momentum (analogous to work being the change in energy): $\vec{J} = \Delta\vec{p} = \int \vec{F}_{net}dt$
  - If no/small impulse, then linear momentum is conserved; an approximate is okay
  - Generally, for collisions linear momentum is conserved (especially for hard/bouncy objects, because then dt is small and the impulse is small): elastic collision
    - **Conservation of linear momentum does not imply conservation of (kinetic) energy, nor vv**
- Center of mass reference frame
  - For an elastic collision, velocities/linear momenta of objects relative to COM reverse (useful for solving for velocities without using energy formulas)
  - What are the symmetries of this object? (for COM)
  - Is linear momentum conserved? Is it approximately conserved? I.e., is there an impulse on the system? Is it large or can it be approximated away?

  - In questions where both kinetic energy and linear momentum are conserved (elastic collisions), may need to use both conservation of linear momentum and conservation of energy formulas to solve (e.g., Section C Quiz 5)
  - Does the problem need to be broken up into multiple phases? (i.e., is linear momentum conserved in part of the collision/process like in the ballistic pendulum problem?)
- Significant Problems
  - Section C Quiz 5: (involves both conservation of energy and conservation of linear momentum) Ch9 #69
  - Person, dog on boat walking to opposite sides
  - Rocket problem: moodle
  - Astronaut game / cat-like thing on a sled: Ch9 #136
  - Ballistic pendulum: Ch9 Sample problem 9-9