

## Moment of Inertia Lab

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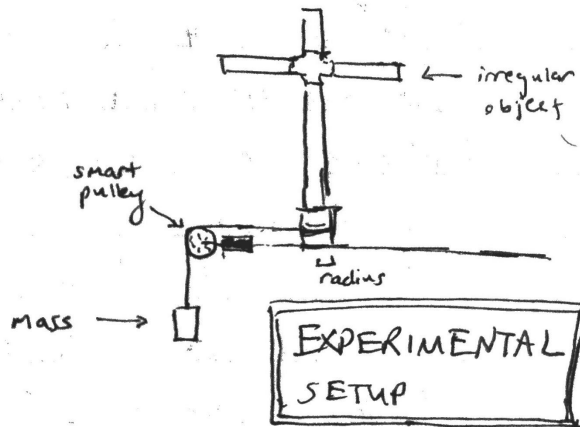
### Introduction

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Purpose: The purpose of this lab is to experimentally determine the moment of inertia (I) of an irregular object.

Procedure: We chose to determine the moment of inertia of a PVC pipe construction (an irregular shape) by using the calculating torque by finding the tension force caused by the measured downwards acceleration. To do this, we first determined the radius of the object by measuring the circumference of the base of the irregular object, which the rope was tautly wrapped around, and then dividing by  $2\pi$ . Then we measured the downwards acceleration of a known mass, (which is equal to the tangential acceleration of the PVC construction). We measured the tangential acceleration using a LabQuest smart pulley by finding the slope of the regression of the velocity. Using just these measurements, we were able to determine the moment of inertia using the formulas (listed below) for net force of the falling object, angular acceleration of the object, and the torque equations.

### Experimental Setup:



### Data and Observations

#### Radius

Variables:  $C$  = circumference (m),  $r$  = radius (m)

$$C = 0.129\text{m} \Rightarrow r = \frac{C}{2\pi} = 0.02053\text{m}$$

#### Acceleration of Different Masses

Mass (kg)	Acceleration ( $\frac{m}{s^2}$ )					
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average

0.050	0.01504	0.01491	0.01358	0.01202	0.01291	0.01369
0.100*	0.02679	0.02797	0.02663	0.02730	0.02809	0.02734
0.150	0.04286	0.04037	0.04411	0.04122	0.04379	0.04247

\* an acceleration of 0.16302 was recorded for the 100g mass but discarded as an outlier

## Calculations / Graphs

### Calculation of the Moment of Inertia (I)

Variables:  $\alpha$  = angular acceleration ( $\frac{\text{rad}}{\text{s}^2}$ ),  $F_T$  = tension force (N),  $\tau$  = torque ( $m \cdot N$ ),  $a$  = tangential acceleration (acceleration of falling mass) ( $\frac{\text{m}}{\text{s}^2}$ ),  $I$  = moment of inertia ( $\text{kg} \cdot \text{m}^2$ ),  $g$  = gravitational acceleration ( $\frac{\text{m}}{\text{s}^2}$ ),  $r$  = radius (m)

$$\alpha = \frac{a}{r}, F_T = mg - ma, \tau = I\alpha = F_T \cdot r \Rightarrow I = \frac{F_T \cdot r}{\alpha} = \frac{(mg - ma)r}{\frac{a}{r}} = \frac{(g - a)mr^2}{a} = \left(\frac{g}{a} - 1\right)mr^2$$

Example calculation (for 50g):  $I = \left(\frac{9.8 \text{ m/s}^2}{0.01369 \text{ m/s}^2} - 1\right) \cdot 0.050 \text{ kg} \cdot (0.0253 \text{ m})^2 = 0.0150 \text{ kg} \cdot \text{m}^2$

### Moments of Inertia for Different Masses

Mass (m, in kg)	Average Tangential Velocity (a, in $\frac{\text{m}}{\text{s}}$ )	Radius (r, in m)	Moment of Inertia (I, in $\text{kg} \cdot \text{m}^2$ )
0.050	0.01369	0.02053	0.0150
0.100	0.02734	0.02053	0.0150
0.150	0.04247	0.02053	0.0145
			<b>Mean I: 0.0148</b>

### Whiteboard

**Torque and Rotation Lab:**

Mass (kg)	Acc. ( $\text{m/s}^2$ )	$\alpha$ ( $\text{rad/sec}^2$ )	$F_T$ (N)	I ( $\text{kgm}^2$ )
0.050	0.01369	0.668	0.489	0.0150
0.100	0.02734	1.335	0.977	0.0150
0.150	0.04247	2.072	1.464	0.0145

Diagram:  $r = 0.0205 \text{ m}$

Ex)  $0.050(9.8) - 0.05(0.01369) = 0.489 \text{ N} = F_T$

Calculations:  
 $mg - ma = F_T$   
 $x = \frac{a}{r}$   
 $\tau = F_T \cdot r = I\alpha \Rightarrow I = \frac{F_T \cdot r}{\alpha}$

$I = \frac{(0.489 - 0.0205)}{0.668} = 0.0150 \text{ kg} \cdot \text{m}^2$

Conclusion: The moment of inertia of the object is roughly a constant 0.0148  $\text{kg} \cdot \text{m}^2$  and is not affected by the force.

*Jon, Maddie, David + Emily*

## Results and Conclusion

### Potential Errors

At first, we had our LabQuest in the wrong setting for the velocity measurements. We had it set to the default setting (picket fence), instead of the ten-spoke wheel. We only noticed this after collecting all of our data. To avoid re-collecting all of the data, we conjectured that there would be a proportional relationship between our measured velocities and the true velocity, in which the proportionality constant is the ratio of the distance between the fence posts and the distance between the wheel spokes. We tested this by doing one trial on each mass and found that there was indeed a proportional relationship of about  $0.22 (\pm 0.01)$ . All of our data was multiplied by this constant to get our final values, which are displayed in the above tables. Because this seems to make sense (there should be a proportional relationship between the speeds), our data were very precise, and our calculated moment of inertia is in the same range as the other groups', we are pretty confident that it should still work.

Another potential error is that we ignored one measured acceleration value from the 100g mass ( $0.16302 \frac{m}{s^2}$ ) because it seemed to be a far outlier that was likely caused by human error caused by accidentally pulling down the string and increasing the acceleration.

Because our values were so close to one another (our three calculated moments of inertia ranged by only  $0.0005kg \cdot m^2$ ) and fell within the class range of  $0.01kg \cdot m^2$  to  $0.02kg \cdot m^2$ , we are fairly confident with our results. A possible factor that may have affected our results is inertia, which may have caused a lower acceleration and slightly lower moment of inertia in our 150g mass trial, but it was most likely insignificant because our calculated values were so consistent between the different masses and accelerations.

### Conclusion

We conclude that the moment of inertia of this irregular object (the PVC pipe construction) is roughly  $0.0148kg \cdot m^2$ , and that this value is not affected by the amount of torque applied to it (in our case, it was not affected by the amount of falling mass pulling on it).