# **Theorem List**

Know how to state, prove, and use the following (unless otherwise indicated):

- \*\* : don't need to know (we didn't go over in class)
- \* : don't need to prove
- 1. All right angles ≅
- 2. All straight angles ≅
- 3. \*\*If a conditional statement is true, then its contrapositive is also true. (given  $p \Rightarrow q, \neg q \Rightarrow \neg p$ )
- 4. SSAC
- 5. SCAC
- 6. CSAC
- 7. CCAC
- 8. Addition property (segment + ≅ segments)
- 9. Addition property (angle  $+ \cong$  angles)
- 10. Addition property (≅ segments + ≅ segments)
- 11. Addition property (≅ angles + ≅ angles)
- 12. Subtraction property (≅ segments/angles segment/angle)
- 13. Subtraction property (≅ segments/angles ≅ segments/angles)
- 14. Multiplication property (like multiples of segments/angles)
- 15. Division property (like divisions of segments/angles)
- 16. Transitive property (2 segments/angles ≅ *same* segment/angle)
- 17. Transitive property (2 segments/angles ≅ ≅ segments/angles)
- 18. VAT
- 19. All radii of a circle ≅
- 20. If two sides of a triangle are  $\cong$ , the angles opposite the sides are  $\cong$  (and inverse\*\*)
- 21. If two angles of a triangle ≅, the sides opposite the angles ≅ (and inverse\*\*)
- 22. Midpoint formula:  $M = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$  (M = median; x<sub>1</sub>, y<sub>1</sub> are coordinates of first point; x<sub>2</sub>, y<sub>2</sub> are coordinates of second point)
- 23. If two angles supplementary and  $\cong$ , then they are right angles
- 24. If 2 points equidistant from the endpoints of a segment, then they determine the perpendicular bisector of that segment
- 25. If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of that segment
- 26. If two (non-vertical) lines are parallel, then their slopes are equal
- 27. If two (non-vertical) lines have the same slope, then they are parallel
- 28. If two lines are perpendicular (and neither is vertical), then each line's slope is the opposite reciprocal of the other's
- 29. If a line's slope is the opposite reciprocal of another line's slope, the two lines are perpendicular
- 30. Remote exterior angle inequality
- 31. Alternate interior angles  $\cong \Rightarrow \parallel$  lines
- 32. Alternate exterior angles  $\cong \Rightarrow \parallel$  lines
- 33. Corresponding angles  $\cong \Rightarrow \parallel$  lines
- 34. SSIS  $\Rightarrow$  || lines
- 35. SSES  $\Rightarrow \parallel$  lines
- 36. If two coplanar lines are perpendicular to a third line, they are  $\parallel$
- 37. || lines  $\Rightarrow \cong$  alternate interior angles

- 38. || lines cut by transversal  $\Rightarrow$  any pair of angles formed are  $\cong$  or supplementary
- 39. || lines  $\Rightarrow$  = alternate exterior angles
- 40. || lines  $\Rightarrow \cong$  corresponding angles
- 41.  $\parallel$  lines  $\Rightarrow$  SSIS
- 42.  $\parallel$  lines  $\Rightarrow$  SSES
- 43. In a plane, if a line is perpendicular to one of two parallel lines, it is perpendicular to the other.
- 44. **\*\***Transitive property (parallel lines)
- 45. \*\*A line and a point not on the line determine a plane
- 46. \*\*Two intersecting lines determine a plane
- 47. \*\*Two parallel lines determine a plane
- 48. **\*\***If a plane is perpendicular to two distinct lines that lie in a plane and that pass through its foot, then it is perpendicular to the plane
- 49. \*\*If a plane intersects two parallel planes, the lines of intersection are parallel
- 50. Triangle sum
- 51. Remote exterior angle sum
- 52. Midline theorem (midline of a triangle is <sup>1</sup>/<sub>2</sub> length of and parallel to side it doesn't intersect)
- 53. No-choice
- 54. AAS ≅
- 55.  $S_i = 180^\circ * (n-2)$
- 56.  $S_e = 360^\circ$
- 57. d = (n \* (n 3)) / 2
- 58. exterior angle of equiangular polygon  $= 360^{\circ} / n$
- 59. MEP
- 60. MER
- 61. The ratio of the perimeters of two similar polygons = ratio of any pair of corresponding sides
- 62. AA
- 63. SSS ~
- 64. SAS ~
- 65. Side-splitter
- 66. If three or more || lines intersected by two transversals, the || lines divide the transversals proportionally
- 67. Angle bisector theorem
- 68. Altitude to hypotenuse
- 69. Pythagorean theorem
- 70.  $a^2 + b^2 = c^2 \Rightarrow$  right triangle (converse of Pythagorean theorem), with c as the longest side and the angle opposite to c being the right angle
- 71. distance =  $\sqrt{(\Delta x)^2 + (\Delta y)^2}$
- 72. In a 30°-60°-90° triangle, the sides opposite these angles are x,  $x\sqrt{3}$ , and 2x, respectively
- 73. In a 45°-45°-90° triangle, the sides opposite these angles are x, x, and  $x\sqrt{2}$ , respectively
- 74. A radius perpendicular to a chord bisects that chord
- 75. A radius that bisects a chord is perpendicular to that chord
- 76. The perpendicular bisector of a chord passes through the center of a circle
- 77. If two chords equidistant from the center, then they are ≅
- 78. If two chords ≅, then they are equidistant from the center
- 79. (79-84)  $\cong$  central angles  $\Leftrightarrow \cong$  arcs  $\Leftrightarrow \cong$  chords
- 85. Two-tangent theorem (2 tangents to same point are  $\cong$ )
- 86. Inscribed angle =  $\frac{1}{2}$  intercepted arc
- 87. Chord-chord angle = mean of intercepted arcs ( $\frac{1}{2}$  sum of intercepted arcs)

- 88. Secant-secant, secant-tangent, tangent-tangent angle =  $\frac{1}{2}$  difference of intercepted arcs
- 89. If two inscribed or tangent-chord angles intercept the same arc, then they are ≅
- 90. If two inscribed or tangent-chord angles intercept ≅ arcs, then they are ≅
- 91. An angle inscribed in a semicircle is a right angle
- 92. \*\*The sum of the measures of a tangent-tangent angle and its minor arc is 180°
- 93. If a quadrilateral is inscribed in a circle, its opposite angles are supplementary
- 94. If a parallelogram is inscribed in a circle, it must be a rectangle
- 95. Chord-chord power theorem
- 96. Tangent-secant power theorem
- 97. Secant-secant power theorem
- 98. \*\*The length of an arc is (AB being the arc, d being the diameter, and AB measured in degrees):  $(\frac{mAB}{360})\pi d$
- 99. Area<sub>square</sub> =  $s^2$  (s = side length)
- 100. Area<sub>parallelogram</sub> = bh (b = base; h = height)
- 101. Area<sub>triangle</sub> =  $\frac{1}{2}bh$
- 102. Area<sub>trapezoid</sub> =  $\frac{1}{2}h(b_1 + b_2)$
- 103.\*\*Median<sub>trapezoid</sub> =  $\frac{1}{2}(b_1 + b_2)$
- 104.\*\*Area<sub>tranezoid</sub> = Mh (M = median; this is a combination of theorems 102 and 103)
- 105. Area<sub>kite/rhombus</sub> =  $\frac{1}{2}d_1d_2$  (d<sub>1</sub> and d<sub>2</sub> = diagonals)
- 106. Area <sub>equilateral triangle</sub> =  $\frac{s^2\sqrt{3}}{4}$
- 107. Area<sub>regular polygon</sub> =  $\frac{1}{2}$ ap (a = apothem; p = perimeter)
- 108.\*\* Area  $_{sector} = (\frac{mAB}{360})\pi r^2$  (mAB is arc measure in degrees; r = radius)
- 109.\*\*Similar figures theorem: If two figures are similar, then the ratio of their areas equals the square of the ratio of their corresponding segments:  $\frac{A_1}{A_2} = (\frac{S_1}{S_2})^2 (A_1, A_2 = \text{areas}; S_1, S_2 = \text{two corresponding sides})$
- 110. A median of a triangle divides the triangle into two triangles with equal areas.
- 111.\*\*Hero's formula (skipped)
- 112.\*\*Brahmagupta's formula (skipped)
- 113.LSA<sub>cylinder</sub> =  $2\pi rh$
- 114.LSA<sub>cone</sub> =  $\pi rl$
- 115.  $V_{rectangular box} = Bh (B = base area; h = height)$
- $116. V_{\text{prism}} = Bh$
- $117. V_{\text{cylinder}} = \pi r^2 h$
- 118.\*\*V<sub>prism/cylinder</sub> = Ch (C = cross-section (parallel to base) area; same as base)
- 119. \* $V_{pyramid} = \frac{1}{3}Bh$
- $120.*V_{cone} = \frac{1}{3}\pi r^2 h$
- 121.\*\*In a pyramid or a cone, the ratio of the area of a cross section to the area of the base equals the square of the ratio of the figures' respective distances from the vertex.
- $122.*V_{\text{sphere}} = \frac{4}{3}\pi r^2$

## Postulates / Properties

#### Postulates are "obvious truths" — inferrable

- Two points determine a line/ray/segment
- A line/ray/segment determines two points
- Three points determine a plane
- A plane determines three points

- Two lines intersect at a point
- Two planes intersect at a line
- If a line is on a plane, then all points on the line must be on the plane
- If two things = each same thing, then they equal each other
- Any segment or angle is  $\cong$  to itself (reflexive property)
- SSS ≅
- SAS ≅
- ASA ≅
- HL
- A line segment is the shortest distance between two points
- Parallel postulate
- If a line intersects a plane not containing it, then the intersection is exactly one point
- AAA ~
- Tangent is perpendicular to radius drawn to the point of contact
- If a line is perpendicular to a radius at its outer endpoint (on the circle), then it is tangent to the circle
- $C_{circle} = \pi d$
- $A_{circle} = \pi r^2$
- $A_{sphere} = 4\pi r^2$
- $V_{rectangular prism} = lwh$

#### **Properties of Geometry**

- Addition, subtraction, multiplication, division properties of algebra
- Addition, subtraction, multiplication, division properties of geometry
- Transitive property
- Substitution property
- Reflexive property
- Properties of quadrilaterals
  - Parallelograms

- Rectangle
  - Rhombus
    - Square
- Kites (<sup>1</sup>/<sub>2</sub> properties of rhombi)
- Trapezoids
  - Isosceles trapezoids

## Definitions

View this <u>Quizlet set</u> (has most of the theorems as well).

### Miscellaneous

- All definitions reversible; some theorems are
- Process of proving a theorem (including thought bubbles!)
- Probability
  - combinations and permutations
  - favorable choices vs. total
- Detour proofs

- Constructions
  - Perpendicular bisector
  - ≅ angles
  - ≅ segments
  - || lines
- Slope
- Indirect Proofs
- Transformations
- Geometric mean / mean proportional
- Indirect measurement
- Angle of incidence, reflection
- Right angle trigonometry
- Law of sines/cosines and derivations
- Walk-around problem
- Three cases (three proofs) of theorem 86 (inscribed angle =  $\frac{1}{2}$  intercepted arc)
- LSA vs. TSA
  - slant height and altitudes
  - great circle
  - "red rice experiments"
- Proportional analysis (unit conversions)